EDISON

Electromagnetic Design of
flexIble SensOrs

FNP Team-Tech

Report 2. Results of analysis

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1 Introduction

The aim of the report is showing the results of analysis for investigated structure which are homogenous open waveguides with diffrent cross-sections.The idea of the analysis is based on the direct field matching technique involving the usage of projection of the fields at the boundary on a fixed set of orthogonal basis functions.

Two regions of investigation can be distinguished in the structure: region I, located inside the waveguide, and region II, outside. The z components of the electric and magnetic fields in both regions have the following form (suppressing $e^{j\omega t}$ time dependence):

$$F_z^{\mathrm{I}} = \sum_{m=-M}^{M} A_m^F J_m(\kappa_{\mathrm{I}}\rho) e^{jm\phi} e^{-\gamma z} \tag{1}$$

$$F_z^{\mathrm{II}} = \sum_{m=-M}^{M} B_m^F H_m^{(2)}(\kappa_{\mathrm{II}}\rho) e^{jm\phi} e^{-\gamma z} \tag{2}$$

where $F = \{E, H\}$, $\kappa_i^2 = \omega^2\mu_i\varepsilon_i + \gamma^2$ for $i = \{\mathrm{I}, \mathrm{II}\}$, ω is the angular frequency, γ is the mode propagation coefficient, $J_m(\cdot)$ and $H_m^{(2)}(\cdot)$ are Bessel and Hankel functions, respectively, of order m and A_m^F and B_m^F are unknown field coefficients. The utilization of the Hankel function of the second kind satisfies Sommerfeld's radiation condition (representing the outward-traveling wave). Due to the assumed field representation in (1) and (2), only convex shapes of the waveguide can be analyzed. The other components of the electric and magnetic fields (E_ϕ, E_ρ, H_ϕ and H_ρ) can be derived from Maxwell's equations.

In order to determine the mode propagation coefficients we need to satisfy the continuity conditions for the tangential field components on the guide surface. Describing the surface of the guide by functions $\rho = \varrho(s)$ and $\phi = \varphi(s)$, where s is the curvilinear coordinate that follows the surface, the continuity conditions for tangential components can be written as follows:

$$F_z^{\mathrm{I}}(\varrho(s), \varphi(s), z) = F_z^{\mathrm{II}}(\varrho(s), \varphi(s), z) \tag{3}$$

$$F_t^{\mathrm{I}}(\varrho(s), \varphi(s), z) = F_t^{\mathrm{II}}(\varrho(s), \varphi(s), z) \tag{4}$$

where $F_t^{(\cdot)}(\cdot) = (\sin\varphi\cos\alpha - \cos\varphi\sin\alpha)F_\rho^{(\cdot)}(\cdot) + (\cos\varphi\cos\alpha + \sin\varphi\sin\alpha)F_\phi^{(\cdot)}(\cdot)$ and $\alpha = \alpha(s)$ is an angle between the x-axis and the normal outgoing vector $\vec{N}$ to the cylinder surface.

We can construct the matrix by using the equations. When the matrix is well known the problem is reduced to finding the roots of the determinant. A complex root finding algorithm based on delaunay triangulation is utilized to find the propagation coefficients of the investigated guides [?].

2 Ellitical fiber

Several guided and leaky modes for various ellipticities of the fiber are calculated and the results are compared with the analytical ones. The cross-section of the guide is illustrated in Fig. 1. The calculated propagation coefficients for different ratios $k = \frac{a}{b}$, maintaining constant area of the guide cross-section. The calculation were performed by selecting $M = 7$ mode expansion functions, and the integrals were evaluated with the use of the trapezoidal rule, with $P = 180$ points evenly covering the boundary contour. Such a choice results from convergence analysis, which is presented in table 2.

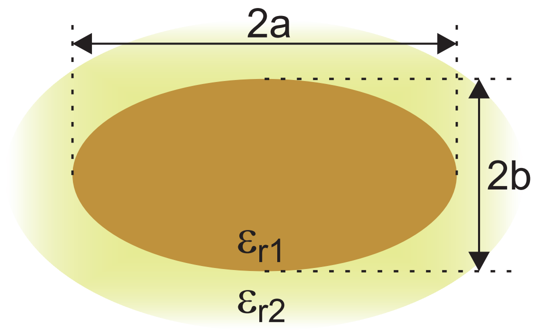

Figure 1: The geometry of the ellitical fiber cross-section. For $k = 1$, $a = b = 0.5$ μm. The material parameters: $\varepsilon_{r1} = 8.41$, $\varepsilon_{r2} = 2.4025$. Frequency corresponds to wavelength $\lambda_0 = 3$ μm in vacuum.

Table 2: Convergence of the method for the example from Fig. 1 (values of propagation coefficients of TM_{01} mode for $k = 3$). Percentage error in bracket.

M	$P = 90$	$P = 180$	$P = 360$	$P = 720$
5	1.7411 (-0,9755%)	1.7417 (-0,9414%)	1.7418 (-0,9357%)	1.7418 (-0,9357%)
6	1.7521 (-0,3653%)	1.7496 (-0.5075%)	1.7491 (-0.5359%)	1.7490 (-0.5416%)
7	1.7522 (-0.3596%)	1.7496 (-0.5075%)	1.7491 (-0.5359%)	1.7490 (-0.5416%)
8	1.7405 (-1.0250%)	1.7518 (-0.3824%)	1.7526 (-0.3369%)	1.7528 (-0.3255%)
9	– (–)	1.7510 (-0.4279%)	1.7527 (-0.3312%)	1.7527 (-0.3312%)
10	– (–)	– (–)	1.7545 (-0.2289%)	1.7562 (-0.1322%)

3 Ellitical fiber with lossy material

The secound structure is ellitical fiber as in the previous section. The difference between last structure is the lossy material in the core of fiber. The cross-section of the guide is illustrated in Fig. 1. The calculated propagation coefficient for ratio $k = 2$ and for diffrent loss coefficients. The calculation were performed by selecting $M = 5$ mode expansion functions, and the integrals were evaluated with the use of the trapezoidal rule, with $P = 720$ points evenly covering the boundary contour.

4 Square with rounded corners fiber

The third structure is a dielectric fiber with square cross-section (rounded corners) with the same material parameters as in the previous example. The analysis is performed again on a single frequency in function of radius of rounded corners as illustrated in Fig. 4. The calculated propagation coefficients for different corner radii, again maintaining constant area of the guide cross-section, is presented in Fig. 5. The calculation were performed by selecting $M = 7$ and $P = 360$. The convergence analysis is presented in table 3.

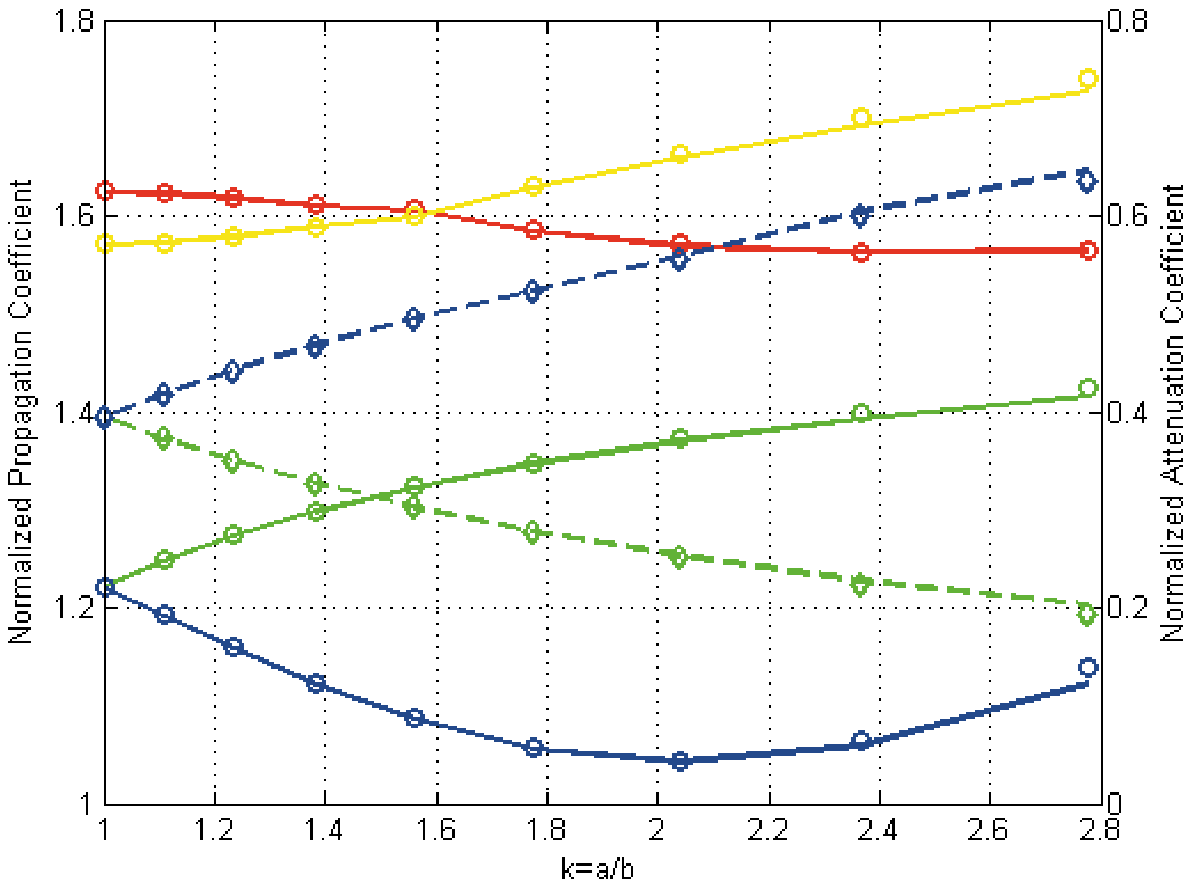

Figure 2: The propagation coefficients for the elliptical fiber in function of ratio $k = \frac{a}{b}$ with parameters from Fig. 1.

Table 3: Convergence of the method for the example from Fig. 4 (values of propagation coefficients).

M	$P = 90$	$P = 180$	$P = 360$	$P = 720$
5	1.5763	1.5764	1.5764	1.5765
6	1.5763	1.5765	1.5765	1.5765
7	1.5763	1.5764	1.5764	1.5765
8	1.5779	1.5781	1.5781	1.5781
9	1.5779	1.5781	1.5781	1.5781
10	1.5779	1.5781	1.5781	1.5781

5 Triangular with rounded corners fiber

The last example considers the triangular fiber with rounded corners depicted in Fig. 6. The calculated propagation coefficients for different corner radii, maintaining constant area of the guide cross-section,

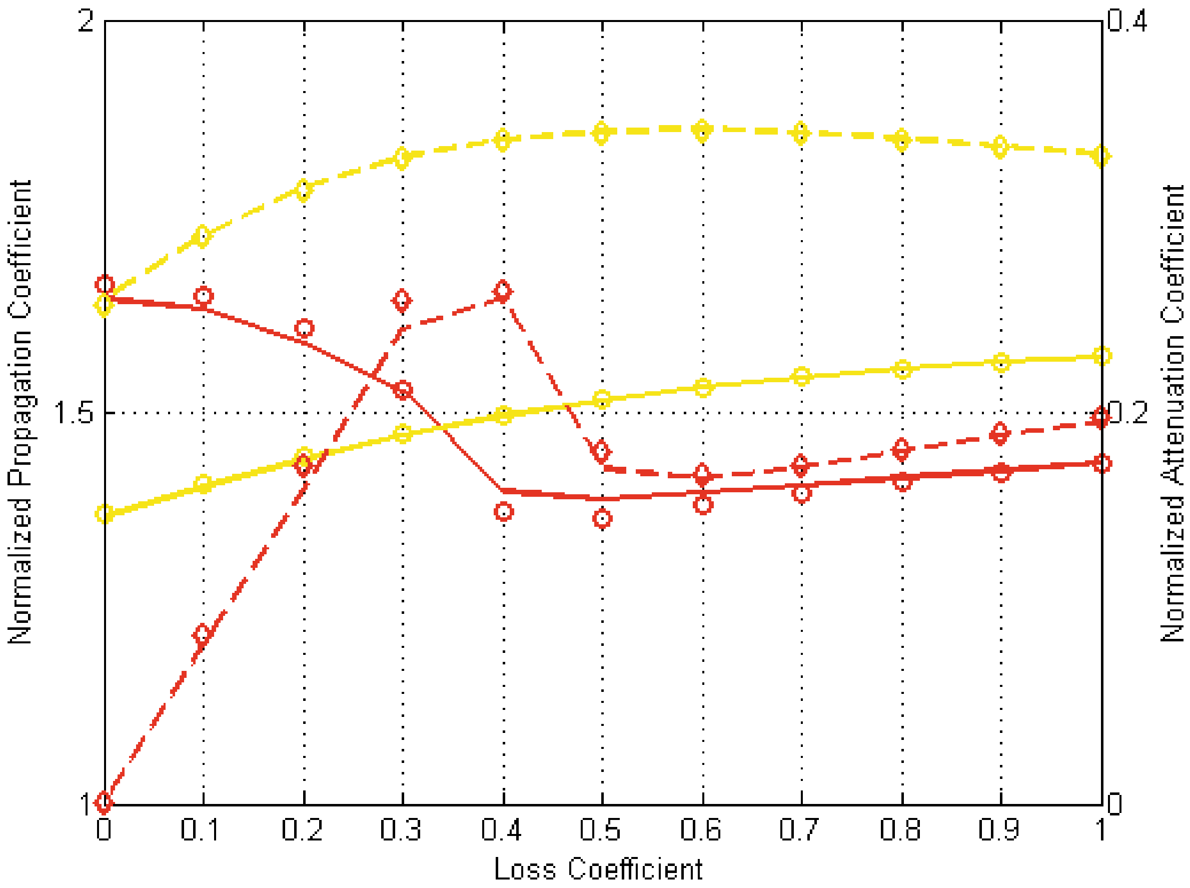

Figure 3: The propagation coefficients for the elliptical fiber in function of loss coefficients with parameters from Fig. 1 and $k = 2$.

Table 4: Convergence of the method for the example from Fig. 6 (values of propagation coefficients).

M	$P = 90$	$P = 180$	$P = 360$	$P = 720$
5	1.5907	1.5910	1.5910	1.5910
6	1.5959	1.5963	1.5963	1.5963
7	1.5959	1.5963	1.5963	1.5963
8	1.5959	1.5963	1.5963	1.5963
9	1.5983	1.5987	1.5987	1.5987
10	1.5983	1.5987	1.5987	1.5987

is presented in Fig. 7. The calculation were performed by selecting $M = 5$ and $P = 360$. The convergence analysis is presented in table 4.

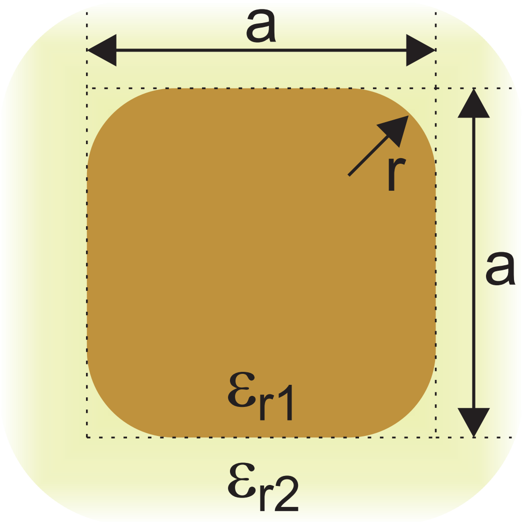

Figure 4: The geometry of the square with rounded corners fiber cross-section. The material parameters as in Fig. 1.

6 Schedule

In next week the results from HFSS will be genereted. The tables and the figures need to be suplemented with HFSS data. The data from algorytm need to be compare with the data from HFSS.

References

[1] P. Kowalczyk, "Complex Root Finding Algorithm Based on Delaunay Triangulation", *ACM Trans. Math. Softw.*, vol. 41,no. 3, pp. 19:1-19:13, Jun. 2015.

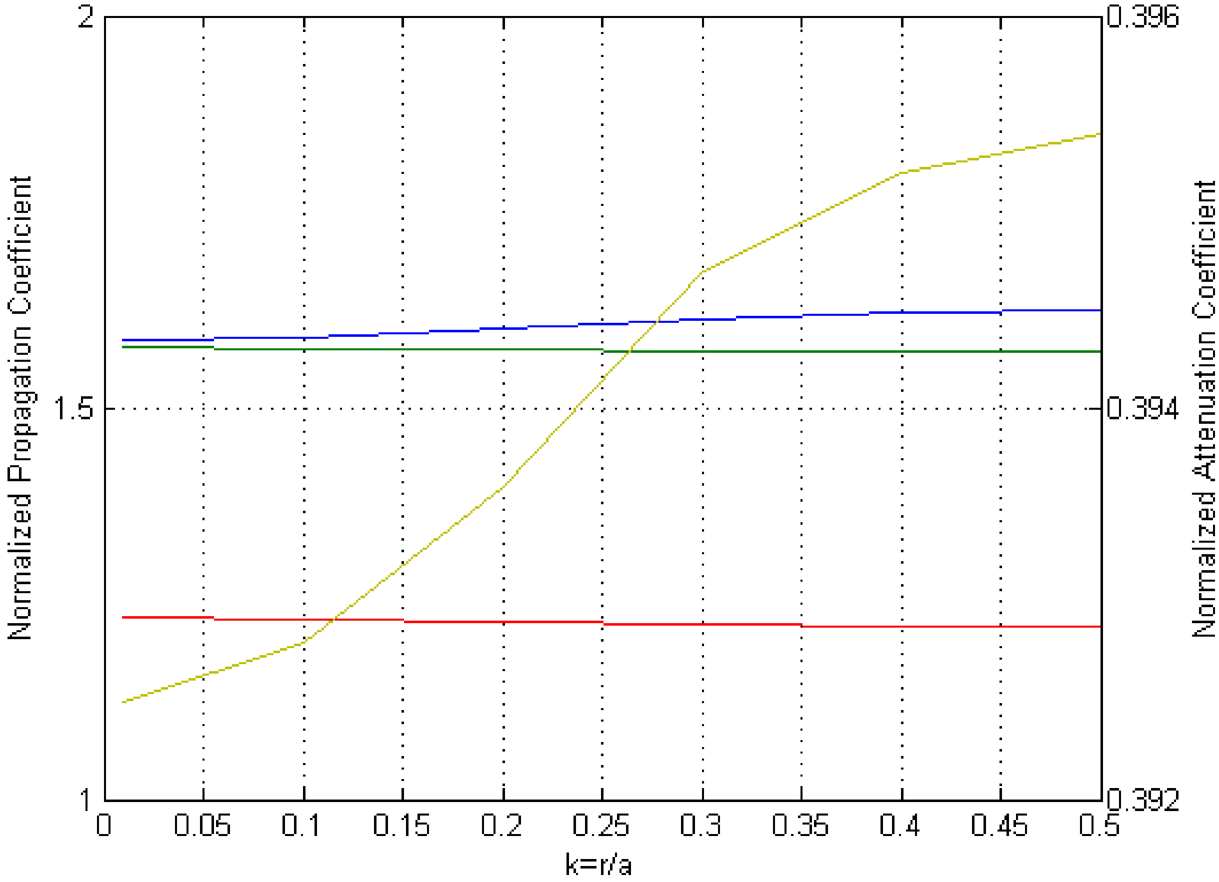

Figure 5: The propagation coefficients for the square fiber in function of radius of rounded corners with parameters from Fig. 4.

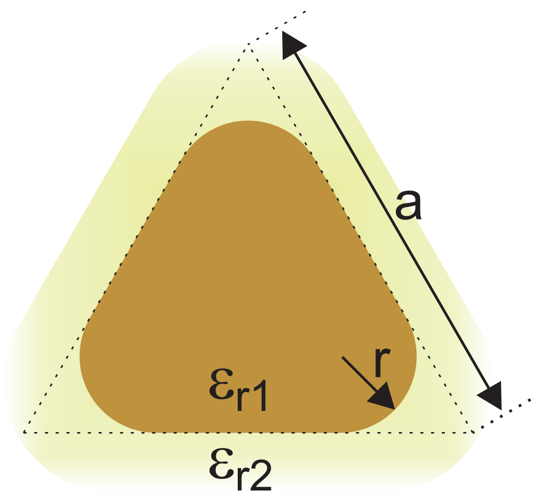

Figure 6: The geometry of the triangular with rounded corners fiber cross-section. The material parameters as in Fig. 1.

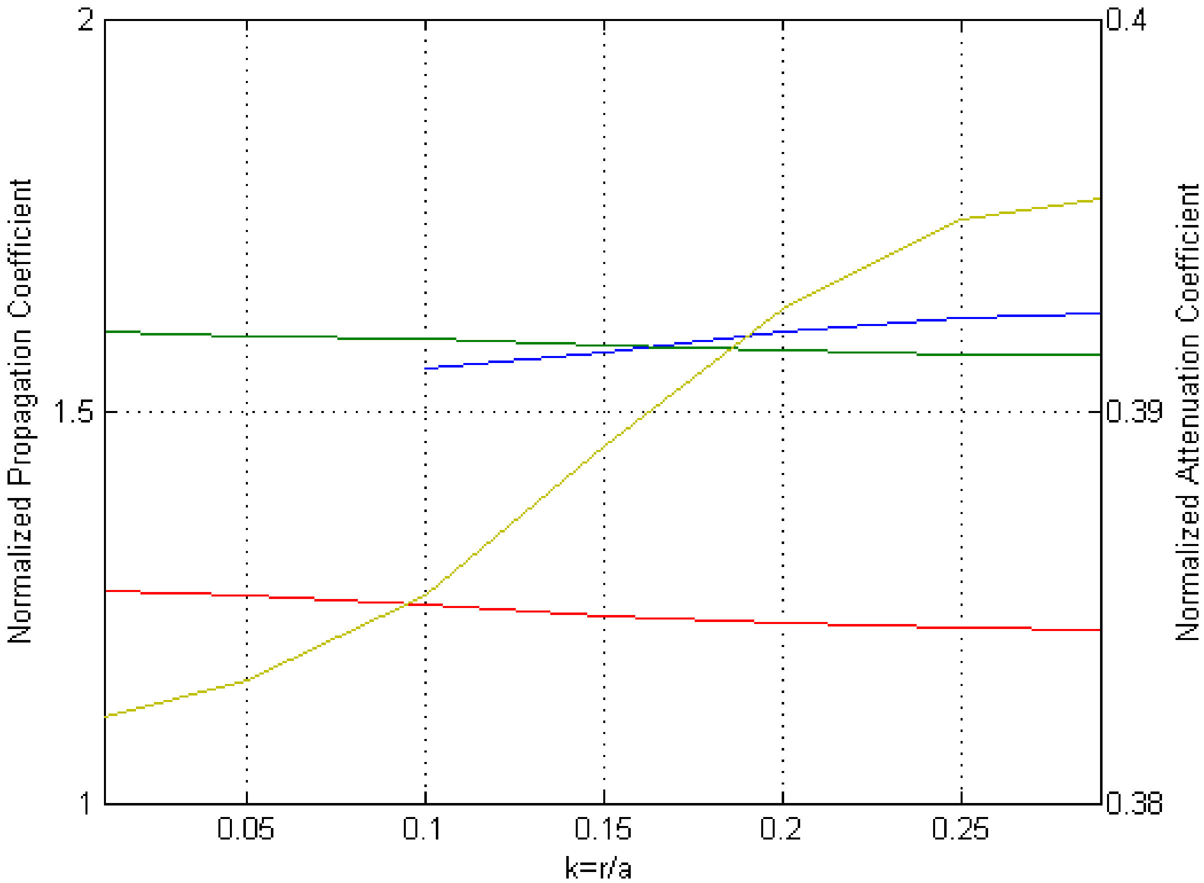

Figure 7: The propagation coefficients for the triangular fiber in function of radius of rounded corners with parameters from Fig. 6.