EDISON

Electromagnetic Design of
flexIble SensOrs

FNP Team-Tech

Report 5. Homogeneous and coaxially loaded cylindrical resonators made of frequency dependent lossy material

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1 Introduction

The aim of the report is to show the results of analysis for two closed structures: homogeneous and coaxially loaded cylindrical resonators presented in the Fig. 1.

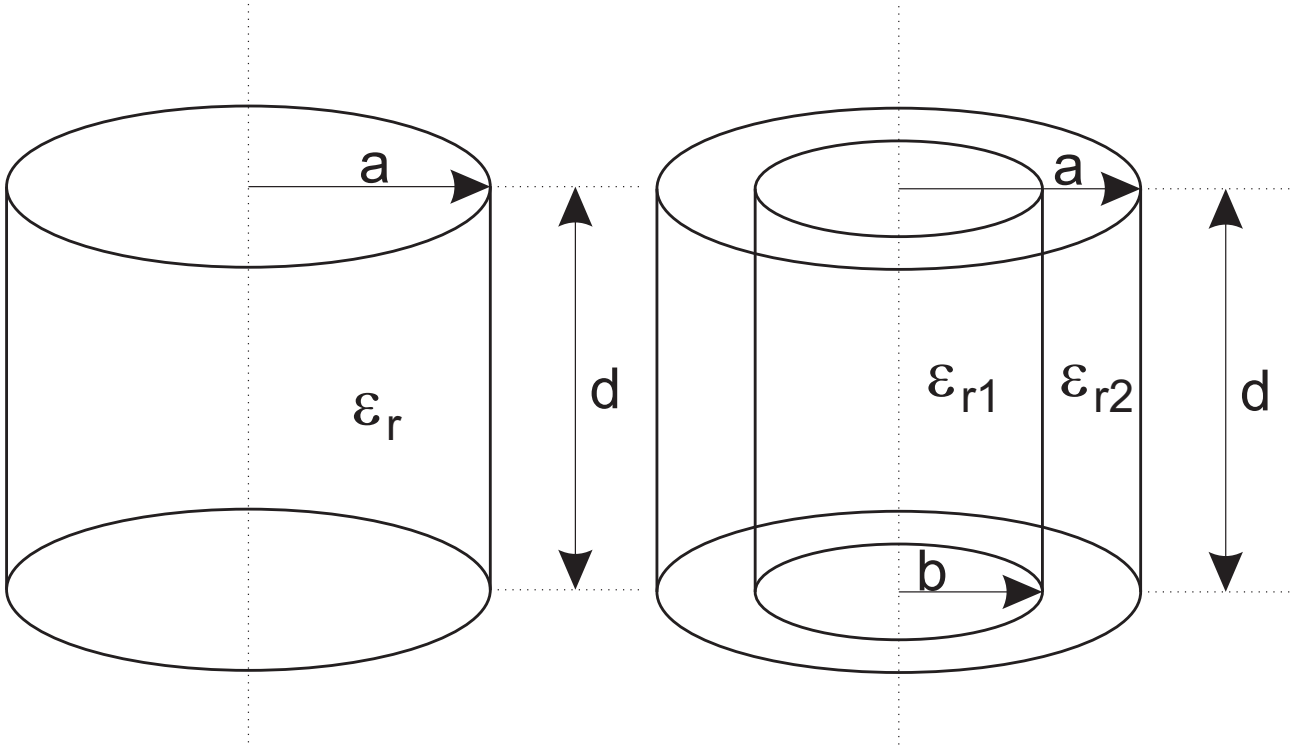

Figure 1: Homogeneous (left) and coaxially loaded (right) cylindrical resonators.

The first considered cylindrical resonator is fully filled with lossy material. The second (coaxially loaded) resonator can be divided into two regions. The first region is a lossy layer and the second one is a vacuum. The permittivity of lossy material can be expressed as $\varepsilon_r = \varepsilon_r' - j\frac{\sigma}{\omega\varepsilon_0}$ where σ is the conductivity of the material, ω is the angular frequency and ε_0 is the permittivity of the vacuum. Results of the analysis are presented in the next sections. For all calculations constants have value $c = 2.99792458 \cdot 10^8 \frac{m}{s}$, $\mu_0 = 4\pi 10^{-7} \frac{H}{m}$ and $\varepsilon_0 = \frac{1}{c^2\mu_0}$. The tolerance of the found frequencies (using root finding algorithm [1]) is 10^{-10} [GHz].

2 Problem formulation

The electric and magnetic fields for cylindrical resonators have the following form (directly from Maxwell's equations):

$$\widehat{E_z} = E_z(\rho)cos(m\phi)cos(\frac{l\pi}{d}z) \tag{1}$$

$$\widehat{E_\rho} = E_\rho(\rho)cos(m\phi)sin(\frac{l\pi}{d}z) \tag{2}$$

$$\widehat{E_\phi} = E_\phi(\rho)sin(m\phi)sin(\frac{l\pi}{d}z) \tag{3}$$

$$\widehat{H_z} = H_z(\rho)sin(m\phi)sin(\frac{l\pi}{d}z) \tag{4}$$

$$\widehat{H_\rho} = H_\rho(\rho)sin(m\phi)cos(\frac{l\pi}{d}z) \tag{5}$$

$$\widehat{H_\phi} = H_\phi(\rho)cos(m\phi)cos(\frac{l\pi}{d}z) \tag{6}$$

where m, n and l refer to the field variation with respect to ϕ, ρ and z, respectively. The boundary conditions can be written as:

$$\frac{\partial\widehat{E_z}(\rho,\phi,d)}{\partial z} = 0 \tag{7}$$

$$\frac{\partial\widehat{E_z}(\rho,\phi,0)}{\partial z} = 0 \tag{8}$$

$$E_\phi(\rho,\phi,d) = 0 \tag{9}$$

$$E_\phi(\rho,\phi,0) = 0 \tag{10}$$

$$E_\phi(a,\phi,z) = 0 \tag{11}$$

$$E_z(a,\phi,z) = 0 \tag{12}$$

2.1 Homogeneous cylindrical resonators

The equations for the homogeneous resonator can be separated due to the symmetry of the structure. Direct consequences of the separation of equations are transverse magnetic and electric modes in the resonator.

For TM_{mnl} modes, the fields can be expressed as:

$$E_\rho(\rho) = -\frac{jl\pi}{d\kappa}AJ'_m(\kappa\rho) \tag{13}$$

$$E_\phi(\rho) = \frac{l\pi m}{d\kappa^2\rho}AJ_m(\kappa\rho) \tag{14}$$

$$E_z(\rho) = AJ_m(\kappa\rho) \tag{15}$$

$$H_\rho(\rho) = -\frac{j\omega\varepsilon m}{\kappa^2\rho}AJ_m(\kappa\rho) \tag{16}$$

$$H_\phi(\rho) = -\frac{\omega\varepsilon m}{\kappa}AJ_m(\kappa\rho) \tag{17}$$

$$H_z(\rho) = 0 \tag{18}$$

and for TE_{mnl} modes :

$$E_\rho(\rho) = \frac{\omega\mu m}{\rho\kappa^2}AJ_m(\kappa\rho) \tag{19}$$

$$E_\phi(\rho) = \frac{\omega\mu}{\kappa}AJ'_m(\kappa\rho) \tag{20}$$

$$E_z(\rho) = 0 \tag{21}$$

$$H_\rho(\rho) = -\frac{jl\pi}{d\kappa}AJ'_m(\kappa\rho) \tag{22}$$

$$H_\phi(\rho) = \frac{jl\pi m}{d\kappa^2\rho}AJ_m(\kappa\rho) \tag{23}$$

$$H_z(\rho) = -jAJ_m(\kappa\rho) \tag{24}$$

where $\kappa^2 = (\frac{2\pi f}{c})^2\varepsilon_r + (\frac{jl\pi}{d})^2$, J_m is the Bessel function, Y_m is the Neumann function. The usage of the boundary conditions reduces the problem to finding zeros of the following expression:
- for TM modes

$$J_m(\kappa a) = 0 \tag{25}$$

- for TE modes

$$J'_m(\kappa a) = 0 \tag{26}$$

2.2 Coaxially loaded cylindrical resonator

If we consider the coaxially loaded cylindrical resonator, then the fields can be expressed as:

$$E_z^I(\rho) = A^I J_m(\kappa_1 \rho) \tag{27}$$

$$E_\phi^I(\rho) = \frac{l\pi m}{d\kappa_1^2 \rho} A^I J_m(\kappa_1 \rho) + \frac{j\omega\mu_1}{\kappa_1} C^I J'_m(\kappa_1 \rho) \tag{28}$$

$$H_z^I(\rho) = C^I J_m(\kappa_1 \rho) \tag{29}$$

$$H_\phi^I(\rho) = \frac{l\pi m}{d\kappa_1^2 \rho} C^I J_m(\kappa_1 \rho) - \frac{j\omega\varepsilon_1}{\kappa_1} A^I J'_m(\kappa_1 \rho) \tag{30}$$

in the first region and

$$E_z^{II}(\rho) = A^{II} J_m(\kappa_2 \rho) + B^{II} Y_m(\kappa_2 \rho) \tag{31}$$

$$E_\phi^{II}(\rho) = \frac{l\pi m}{d\kappa_2^2 \rho} A^{II} J_m(\kappa_2 \rho) + \frac{l\pi m}{d\kappa_2^2 \rho} B^{II} Y_m(\kappa_2 \rho) + \frac{j\omega\mu_2}{\kappa_2} C^{II} J'_m(\kappa_2 \rho) + \frac{j\omega\mu_2}{\kappa_2} D^{II} Y'_m(\kappa_2 \rho) \tag{32}$$

$$H_z^{II}(\rho) = C^{II} J_m(\kappa_2 \rho) + D^{II} Y_m(\kappa_2 \rho) \tag{33}$$

$$H_\phi^{II}(\rho) = \frac{l\pi m}{d\kappa_2^2 \rho} C^{II} J_m(\kappa_2 \rho) + \frac{l\pi m}{d\kappa_2^2 \rho} D^{II} Y_m(\kappa_2 \rho) + \frac{j\omega\mu_2}{\kappa_2} A^{II} J'_m(\kappa_2 \rho) + \frac{j\omega\mu_2}{\kappa_2} B^{II} Y'_m(\kappa_2 \rho) \tag{34}$$

in the second region. The resonance frequencies for this structure can be found also from boundary conditions (7-12) and continuity of the fields at the connection of the regions, by finding the roots of the following determinant:

$$\begin{vmatrix} J_m(\kappa_1 a) & 0 & -J_m(\kappa_2 a) & -Y_m(\kappa_2 a) & 0 & 0 \\ 0 & J_m(\kappa_1 a) & 0 & 0 & -J_m(\kappa_2 a) & -Y_m(\kappa_2 a) \\ -\frac{j\omega\varepsilon_1}{\kappa_1} J'_m(\kappa_1 a) & \frac{l\pi m}{da\kappa_1^2} J_m(\kappa_1 a) & \frac{j\omega\varepsilon_2}{\kappa_2} J'_m(\kappa_2 a) & \frac{j\omega\varepsilon_2}{\kappa_2} Y'_m(\kappa_2 a) & -\frac{l\pi m}{da\kappa_2^2} J_m(\kappa_2 a) & -\frac{l\pi m}{da\kappa_2^2} Y_m(\kappa_2 a) \\ \frac{l\pi m}{da\kappa_1^2} J_m(\kappa_1 a) & \frac{j\omega\mu_1}{\kappa_1} J'_m(\kappa_1 a) & -\frac{l\pi m}{da\kappa_2^2} J_m(\kappa_2 a) & -\frac{l\pi m}{da\kappa_2^2} Y_m(\kappa_2 a) & -\frac{j\omega\mu_2}{\kappa_2} J'_m(\kappa_2 a) & -\frac{j\omega\mu_2}{\kappa_2} J'_m(\kappa_2 a) \\ 0 & 0 & \frac{l\pi m}{db\kappa_2^2} J_m(\kappa_2 b) & \frac{l\pi m}{db\kappa_2^2} Y_m(\kappa_2 b) & \frac{j\omega\mu_2}{\kappa_2} J'_m(\kappa_2 b) & \frac{j\omega\mu_2}{\kappa_2} J'_m(\kappa_2 b) \\ 0 & 0 & J_m(\kappa_2 b) & Y_m(\kappa_2 b) & 0 & 0 \end{vmatrix} = 0 \tag{35}$$

3 Results

3.1 Homogeneous cylindrical resonator

This section presents results for transverse magnetic and transverse electric modes in the homogeneous cylindrical resonator. The geometrical dimensions used in the analysis are $d = 10\sqrt{2}mm$ and $a = 10mm$ (see Fig.1) and the relative permittivity is $\varepsilon'_r = 10$. The analysed modes are described in the table 1. In the first step the analysis is performed for $\sigma = 0$ and the results are presented in the table 2. The complex resonance frequencies as a function of conductivity (in the range 0 to 100 S/m) are presented in the Fig. 2-4 and shown in the table 2. The initial mesh step in the global root finding algorithm [1] is set to 0.01 [GHz]. In table 4 Q factor of the resonator as a function of σ is presented.

3.2 Coaxially loaded cylindrical resonator

This section demonstrates the results for modes in a coaxially loaded cylindrical resonator. The geometrical dimensions used in the analysis are $d = 10\sqrt{2}mm$, $a = 6.35mm$ and $b = 10mm$ (see Fig.1) and the and the relative permittivities are $\varepsilon_{r2} = 1$ and $\varepsilon'_{r1} = 10$. The analysed modes are described in the table 5. In the first step the analysis is performed for $\sigma = 0$ and the results are presented in the table 2. The complex resonance frequencies as a function of conductivity (in the range 0 to 50 S/m) are presented in the Fig. 5-7 colected in the table 3.2. The results are confronted with the results for the coaxially loaded resonator with perfect electric conductor (see table 8). The initial mesh step in the global root finding algorithm [1] is set to 0.01 [GHz]. In table 7 Q factor of the resonator as a function of σ is presented.

Table 2: Modes presented in Fig.2-4.

mode	color in figure	line style
TE_{011}	red	solid
TE_{012}	green	solid
TE_{111}	black	solid
TE_{112}	cyan	solid
TE_{121}	magenta	solid
TE_{211}	yellow	solid
TE_{212}	blue	dotted
TE_{311}	red	dotted
TE_{411}	green	dotted
TM_{010}	black	dotted
TM_{011}	cyan	dotted
TM_{012}	magenta	dotted
TM_{020}	yellow	dotted
TM_{021}	blue	dashed
TM_{110}	red	dashed
TM_{111}	green	dashed
TM_{112}	black	dashed
TM_{210}	cyan	dashed
TM_{211}	magenta	dashed
TM_{310}	yellow	dashed

Table 3: The resonance frequencies for $\sigma = 0$

	homogeneous	partially loaded	
	$\varepsilon_{r2} = 10$	$\varepsilon_{r2} = 1$	
TM_{010}	3.628477324	3.883856934	TM_{010}
TE_{111}	4.353381158	6.088975322	HE_{111}
TM_{011}	4.939664708	8.193372828	TM_{011}
TE_{211}	5.698345714	9.884743615	HE_{211}
TM_{110}	5.781399895	6.618765916	TM_{110}
TM_{111}	6.682740791	9.884743615	EH_{111}
TE_{011}	6.682740791	7.575026719	HE_{011}
TE_{311}	7.170490584	12.308462125	HE_{311}
TE_{112}	7.256393506	8.439295609	HE_{112}
TM_{012}	7.622572179	10.941790074	TM_{012}
TM_{210}	7.748790347	9.389886432	TM_{210}
TE_{212}	8.134768778	10.796266012	HE_{212}
TM_{020}	8.328869505	10.120293653	TM_{020}
TM_{211}	8.442641268	12.403656676	EH_{211}
TE_{411}	8.695267456	15.220919382	HE_{411}
TE_{121}	8.714608766	11.393804922	HE_{121}
TE_{012}	8.852250769	9.8095604466	HE_{012}
TM_{112}	8.852250769	12.101986836	EH_{112}
TM_{021}	8.978001279	12.187616669	TM_{021}
TM_{310}	9.626591289	12.204096657	TM_{310}

Table 4: The resonance frequencies of the homogeneous resonator as a function of the conductivity.

σ	0.01	0.05
TM_{011}	4.939656531 + 0.008987551i	4.939460297 + 0.044937758i
TM_{011}	4.939656531 + 0.008987551i	4.939460297 + 0.044937758i
TM_{021}	8.977996780 + 0.008987551i	8.977888814 + 0.044937758i
TM_{021}	8.977996780 + 0.008987551i	8.977888814 + 0.044937758i
TM_{010}	3.628466193 + 0.008987551i	3.628199042 + 0.044937758i
TM_{010}	3.628466193 + 0.008987551i	3.628199042 + 0.044937758i
TM_{020}	8.328864655 + 0.008987551i	8.328748275 + 0.044937758i
TM_{020}	8.328864655 + 0.008987551i	8.328748275 + 0.044937758i
TM_{110}	5.781392909 + 0.008987551i	5.781225245 + 0.044937758i
TM_{110}	5.781392909 + 0.008987551i	5.781225245 + 0.044937758i
TM_{210}	7.748785135 + 0.008987551i	7.748660042 + 0.044937758i
TM_{210}	7.748785135 + 0.008987551i	7.748660042 + 0.044937758i
TM_{310}	9.626587093 + 0.008987551i	9.626486401 + 0.044937758i
TM_{310}	9.626587093 + 0.008987551i	9.626486401 + 0.044937758i
TM_{111}	6.682734747 + 0.008987551i	6.682589698 + 0.044937758i
TM_{111}	6.682734747 + 0.008987551i	6.682589698 + 0.044937758i
TM_{211}	8.442636484 + 0.008987551i	8.442521672 + 0.044937758i
TM_{211}	8.442636484 + 0.008987551i	8.442521672 + 0.044937758i
TM_{112}	8.852246206 + 0.008987551i	8.852136707 + 0.044937758i
TM_{112}	8.852246206 + 0.008987551i	8.852136707 + 0.044937758i
TE_{011}	6.682734747 + 0.008987551i	6.682589698 + 0.044937758i
TE_{011}	6.682734747 + 0.008987551i	6.682589698 + 0.044937758i
TE_{012}	8.852246206 + 0.008987551i	8.852136707 + 0.044937758i
TE_{012}	8.852246206 + 0.008987551i	8.852136707 + 0.044937758i
TM_{012}	7.622566880 + 0.008987551i	7.622439716 + 0.044937758i
TM_{012}	7.622566880 + 0.008987551i	7.622439716 + 0.044937758i
TE_{111}	4.353371881 + 0.008987551i	4.353149217 + 0.044937758i
TE_{111}	4.353371881 + 0.008987551i	4.353149217 + 0.044937758i
TE_{121}	8.714604131 + 0.008987551i	8.714492902 + 0.044937758i
TE_{121}	8.714604131 + 0.008987551i	8.714492902 + 0.044937758i
TE_{211}	5.698338626 + 0.008987551i	5.698168519 + 0.044937758i
TE_{211}	5.698338626 + 0.008987551i	5.698168519 + 0.044937758i
TE_{112}	7.256387940 + 0.008987551i	7.256254358 + 0.044937758i
TE_{112}	7.256387940 + 0.008987551i	7.256254358 + 0.044937758i
TE_{311}	7.170484951 + 0.008987551i	7.170349769 + 0.044937758i
TE_{311}	7.170484951 + 0.008987551i	7.170349769 + 0.044937758i
TE_{212}	8.134763813 + 0.008987551i	8.134644655 + 0.044937758i
TE_{212}	8.134763813 + 0.008987551i	8.134644655 + 0.044937758i
TE_{411}	8.695262811 + 0.008987551i	8.695151335 + 0.044937758i
TE_{411}	8.695262811 + 0.008987551i	8.695151335 + 0.044937758i

σ	0.1	0.5
TM_{011}	4.938847013 + 0.089875517i	4.919181559 + 0.449377589i
TM_{011}	4.938847013 + 0.089875517i	4.919181559 + 0.449377589i
TM_{021}	8.977551412 + 0.089875517i	8.966747835 + 0.449377589i
TM_{021}	8.977551412 + 0.089875517i	8.966747835 + 0.449377589i
TM_{010}	3.627364068 + 0.089875517i	3.600542664 + 0.449377589i
TM_{010}	3.627364068 + 0.089875517i	3.600542664 + 0.449377589i
TM_{020}	8.328384574 + 0.0898755178i	8.31673776 + 0.449377589i
TM_{020}	8.328384574 + 0.0898755178i	8.31673776 + 0.449377589i
TM_{110}	5.780701266 + 0.0898755178i	5.76390878 + 0.449377589i
TM_{110}	5.780701266 + 0.0898755178i	5.76390878 + 0.449377589i
TM_{210}	7.748269112 + 0.0898755178i	7.73574893 + 0.449377589i
TM_{210}	7.748269112 + 0.0898755178i	7.73574893 + 0.449377589i
TM_{310}	9.626171733 + 0.089875517i	9.616096902 + 0.449377589i
TM_{310}	9.626171733 + 0.089875517i	9.616096902 + 0.449377589i
TM_{111}	6.682136400 + 0.089875517i	6.667614585 + 0.449377589i
TM_{111}	6.682136400 + 0.089875517i	6.667614585 + 0.449377589i
TM_{211}	8.442162873 + 0.089875517i	8.430673245 + 0.449377589i
TM_{211}	8.442162873 + 0.089875517i	8.430673245 + 0.449377589i
TM_{112}	8.851794511 + 0.089875517i	8.840837260 + 0.449377589i
TM_{112}	8.851794511 + 0.089875517i	8.840837260 + 0.449377589i
TE_{011}	6.682136400 + 0.089875517i	6.667614585 + 0.449377589i
TE_{011}	6.682136400 + 0.089875517i	6.667614585 + 0.449377589i
TE_{012}	8.851794511 + 0.089875517i	8.840837260 + 0.449377589i
TE_{012}	8.851794511 + 0.089875517i	8.840837260 + 0.449377589i
TM_{012}	7.622042313 + 0.089875517i	7.609314450 + 0.449377589i
TM_{012}	7.622042313 + 0.089875517i	7.609314450 + 0.449377589i
TE_{111}	4.352453320 + 0.089875517i	4.330125551 + 0.449377589i
TE_{111}	4.352453320 + 0.089875517i	4.330125551 + 0.449377589i
TE_{121}	8.714145301 + 0.089875517i	8.703014749 + 0.449377589i
TE_{121}	8.714145301 + 0.089875517i	8.703014749 + 0.449377589i
TE_{211}	5.697636902 + 0.089875517i	5.6805988821 + 0.44937758i
TE_{211}	5.697636902 + 0.089875517i	5.6805988822 + 0.44937758i
TE_{112}	7.255836899 + 0.089875517i	7.2424654984 + 0.44937758i
TE_{112}	7.255836899 + 0.089875517i	7.2424654984 + 0.44937758i
TE_{311}	7.169927308 + 0.089875517i	7.1563953915 + 0.44937758i
TE_{311}	7.169927308 + 0.089875517i	7.1563953915 + 0.44937758i
TE_{212}	8.134272276 + 0.089875517i	8.1223471277 + 0.44937758i
TE_{212}	8.134272276 + 0.089875517i	8.1223471277 + 0.44937758i
TE_{411}	8.694802961 + 0.089875517i	8.6836476164 + 0.44937758i
TE_{411}	8.694802961 + 0.089875517i	8.6836476164 + 0.44937758i

σ	1	5
TM_{011}	4.857213867 + 0.898755178i	2.050918244 + 4.493775893i
TM_{011}	4.857213867 + 0.898755178i	2.050918244 + 4.493775893i
TM_{021}	8.932902445 + 0.898755178i	7.772418232 + 4.493775893i
TM_{021}	8.932902445 + 0.898755178i	7.772418232 + 4.493775893i
TM_{010}	3.515407063 + 0.898755178i	1.842705527i
TM_{010}	3.515407063 + 0.898755178i	7.144846260i
TM_{020}	8.280235887 + 0.898755178i	7.012563400 + 4.493775893i
TM_{020}	8.280235887 + 0.898755178i	7.012563400 + 4.493775893i
TM_{110}	5.711114066 + 0.898755178i	3.637384082 + 4.493775893i
TM_{110}	5.711114066 + 0.898755178i	3.637384082 + 4.493775893i
TM_{210}	7.696492121 + 0.898755178i	6.312664260 + 4.493775893i
TM_{210}	7.696492121 + 0.898755178i	6.312664260 + 4.493775893i
TM_{310}	9.584544797 + 0.898755178i	8.513356451 + 4.493775893i
TM_{310}	9.584544797 + 0.898755178i	8.513356451 + 4.493775893i
TM_{111}	6.622028662 + 0.898755178i	4.946210943 + 4.493775893i
TM_{111}	6.622028662 + 0.898755178i	4.946210943 + 4.493775893i
TM_{211}	8.394666801 + 0.898755178i	7.147319064 + 4.493775893i
TM_{211}	8.394666801 + 0.898755178i	7.1473190641 + 4.493775893i
TM_{112}	8.806507980 + 0.898755178i	7.6268159739 + 4.493775893i
TM_{112}	8.806507980 + 0.898755178i	7.6268159739 + 4.493775893i
TE_{011}	6.622028662 + 0.898755178i	4.9462109435 + 4.493775893i
TE_{011}	6.622028662 + 0.898755178i	4.9462109435 + 4.493775893i
TE_{012}	8.806507980 + 0.898755178i	7.6268159739 + 4.493775893i
TE_{012}	8.806507980 + 0.898755178i	7.6268159739 + 4.493775893i
TM_{012}	7.569401941 + 0.898755178i	6.1570759982 + 4.493775893i
TM_{012}	7.569401941 + 0.898755178i	6.1570759982 + 4.493775893i
TE_{111}	4.259597004 + 0.898755178i	3.3792830626i
TE_{111}	4.259597004 + 0.898755178i	5.6082687247i
TE_{121}	8.668139654 + 0.898755178i	7.4666179868 + 4.493775893i
TE_{121}	8.668139654 + 0.898755178i	7.4666179868 + 4.493775893i
TE_{211}	5.627022570 + 0.898755178i	3.5038724429 + 4.493775893i
TE_{211}	5.627022570 + 0.898755178i	3.5038724429 + 4.493775893i
TE_{112}	7.200519831 + 0.898755178i	5.6974753121 + 4.493775893i
TE_{112}	7.200519831 + 0.898755178i	5.6974753121 + 4.493775893i
TE_{311}	7.113942250 + 0.898755178i	5.5876572403 + 4.493775893i
TE_{311}	7.113942250 + 0.898755178i	5.5876572403 + 4.493775893i
TE_{212}	8.084967669 + 0.898755178i	6.7808879432 + 4.493775893i
TE_{212}	8.084967669 + 0.898755178i	6.7808879432 + 4.493775893i
TE_{411}	8.648694425 + 0.898755178i	7.4440348174 + 4.493775893i
TE_{411}	8.648694425 + 0.898755178i	7.4440348174 + 4.493775893i

σ	10	50	100
TM_{011}	1.479169824i	0.272314901i	0.135847573i
TM_{011}	16.495933750i	89.603202971i	179.6151881i
TM_{021}	9.401774149i	88.969539179i	179.301488408i
TM_{021}	7.772418232 + 4.493775893i	8.573329425i	0.449547338i
TM_{010}	0.765007203i	0.146729359i	0.073274776i
TM_{010}	17.210096371i	89.728788513i	179.677760971i
TM_{020}	5.610271831i	0.778591020i	0.386755194i
TM_{020}	12.364831743i	89.096926852i	179.3642805i
TM_{110}	2.1063089937i	0.373450420i	0.186142090i
TM_{110}	15.868794580i	89.502067452i	179.5648936i
TM_{210}	13.540829209i	89.202399751i	179.4163742i
TM_{210}	4.434274365i	0.673118122i	0.334661493i
TM_{310}	8.513356451 + 4.493775893i	3.448937911 + 8.987551787i	0.517040641i
TM_{310}	3.448937911 + 8.987551787i	88.8323020876i	179.233995105i
TM_{111}	14.997299090i	89.375841183i	179.062539585i
TM_{111}	2.977804484i	0.499676689i	0.248793686i
TM_{211}	5.905686192i	0.800201384i	0.397417080i
TM_{211}	12.069417382i	89.075316489i	179.353618666i
TM_{112}	10.541174472i	88.994992560i	179.314023951i
TM_{112}	7.433929102i	0.880525313i	0.437011796i
TE_{011}	14.99729909i	89.375841183i	179.062539585i
TE_{011}	2.977804484i	0.499676689i	0.248793686i
TE_{012}	10.541174472i	88.994992560i	179.314023951i
TE_{012}	7.433929102i	0.880525313i	0.437011796i
TM_{012}	13.74911460i	89.224309622i	179.427207452i
TM_{012}	4.225988966i	0.651208251i	0.323828294i
TE_{111}	1.124717716i	0.211365713i	0.105496231i
TE_{111}	16.85038585i	89.664152160i	179.645539515i
TE_{121}	11.185660334i	89.022424891i	179.327540249i
TE_{121}	6.789443239i	0.853092982i	0.423495497i
TE_{211}	2.037376016i	0.362754344i	0.180827009i
TE_{211}	15.93772755i	89.512763528i	179.570208737i
TE_{112}	14.290460046i	89.285779541i	179.457622483i
TE_{112}	3.684643112i	0.589738332i	0.293413263i
TE_{311}	3.569050895i	0.575767964i	0.286496348i
TE_{311}	0.648388316i	0.125174336i	0.062521746i
TE_{212}	5.166344641i	0.742423038i	0.368902196i
TE_{212}	12.808758932i	89.133094835i	179.382133550i
TE_{411}	11.260965736i	89.0262438520i	179.329422410i
TE_{411}	6.7141378379i	0.849274021i	0.421613336i

Table 5: The Q factor of the homogeneous resonator as a function of conductivity.

σ	0.01	0.05	0.1	0.5	1	5	10	50	100
TM_{011}	274.805	54.959	27.476	5.473	2.702	0.228	0.000	0	0
TM_{011}	274.805	54.959	27.476	5.473	2.702	0.228	0.000	0	0
TM_{021}	499.468	99.892	49.944	9.977	4.970	0.865	0.000	0	0
TM_{021}	499.468	99.892	49.944	9.977	4.970	0.865	0.000	0	0
TM_{010}	201.861	40.369	20.180	4.006	1.956	0.000	0.000	0	0
TM_{010}	201.861	40.369	20.180	4.006	1.956	0.000	0.000	0	0
TM_{020}	463.356	92.670	46.333	9.254	4.607	0.780	0.000	0	0
TM_{020}	463.356	92.670	46.333	9.254	4.607	0.780	0.000	0	0
TM_{110}	321.633	64.325	32.159	6.413	3.177	0.405	0.000	0	0
TM_{110}	321.633	64.325	32.159	6.413	3.177	0.405	0.000	0	0
TM_{210}	431.084	86.215	43.106	8.607	4.282	0.702	0.000	0	0
TM_{210}	431.084	86.215	43.106	8.607	4.282	0.702	0.000	0	0
TM_{310}	535.551	107.109	53.553	10.699	5.332	0.947	0.192	0	0
TM_{310}	535.551	107.109	53.553	10.699	5.332	0.947	0.192	0	0
TM_{111}	371.777	74.354	37.174	7.419	3.684	0.550	0.000	0	0
TM_{111}	371.777	74.354	37.174	7.419	3.684	0.550	0.000	0	0
TM_{211}	469.685	93.936	46.966	9.380	4.670	0.795	0.000	0	0
TM_{211}	469.685	93.936	46.966	9.380	4.670	0.795	0.000	0	0
TM_{112}	492.473	98.493	49.245	9.837	4.899	0.849	0.000	0	0
TM_{112}	492.473	98.493	49.245	9.837	4.899	0.849	0.000	0	0
TE_{011}	371.777	74.354	37.174	7.419	3.684	0.550	0.000	0	0
TE_{011}	371.777	74.354	37.174	7.419	3.684	0.550	0.000	0	0
TE_{012}	492.473	98.493	49.245	9.837	4.899	0.849	0.000	0	0
TE_{012}	492.473	98.493	49.245	9.837	4.899	0.849	0.000	0	0
TM_{012}	424.062	84.811	42.403	8.467	4.211	0.685	0.000	0	0
TM_{012}	424.062	84.811	42.403	8.467	4.211	0.685	0.000	0	0
TE_{111}	242.189	48.435	24.214	4.818	2.370	0.000	0.000	0	0
TE_{111}	242.189	48.435	24.214	4.818	2.370	0.000	0.000	0	0
TE_{121}	484.815	96.962	48.479	9.683	4.822	0.831	0.000	0	0
TE_{121}	484.815	96.962	48.479	9.683	4.822	0.831	0.000	0	0
TE_{211}	317.013	63.401	31.697	6.321	3.130	0.390	0.000	0	0
TE_{211}	317.013	63.401	31.697	6.321	3.130	0.390	0.000	0	0
TE_{112}	403.691	80.737	40.366	8.058	4.006	0.634	0.000	0	0
TE_{112}	403.691	80.737	40.366	8.058	4.006	0.634	0.000	0	0
TE_{311}	398.912	79.781	39.888	7.963	3.958	0.622	0.000	0	0
TE_{311}	398.912	79.781	39.888	7.963	3.958	0.622	0.000	0	0
TE_{212}	452.557	90.510	45.253	9.037	4.498	0.754	0.000	0	0
TE_{212}	452.557	90.510	45.253	9.037	4.498	0.754	0.000	0	0
TE_{411}	483.739	96.747	48.371	9.662	4.811	0.828	0.000	0	0
TE_{411}	483.739	96.747	48.371	9.662	4.811	0.828	0.000	0	0

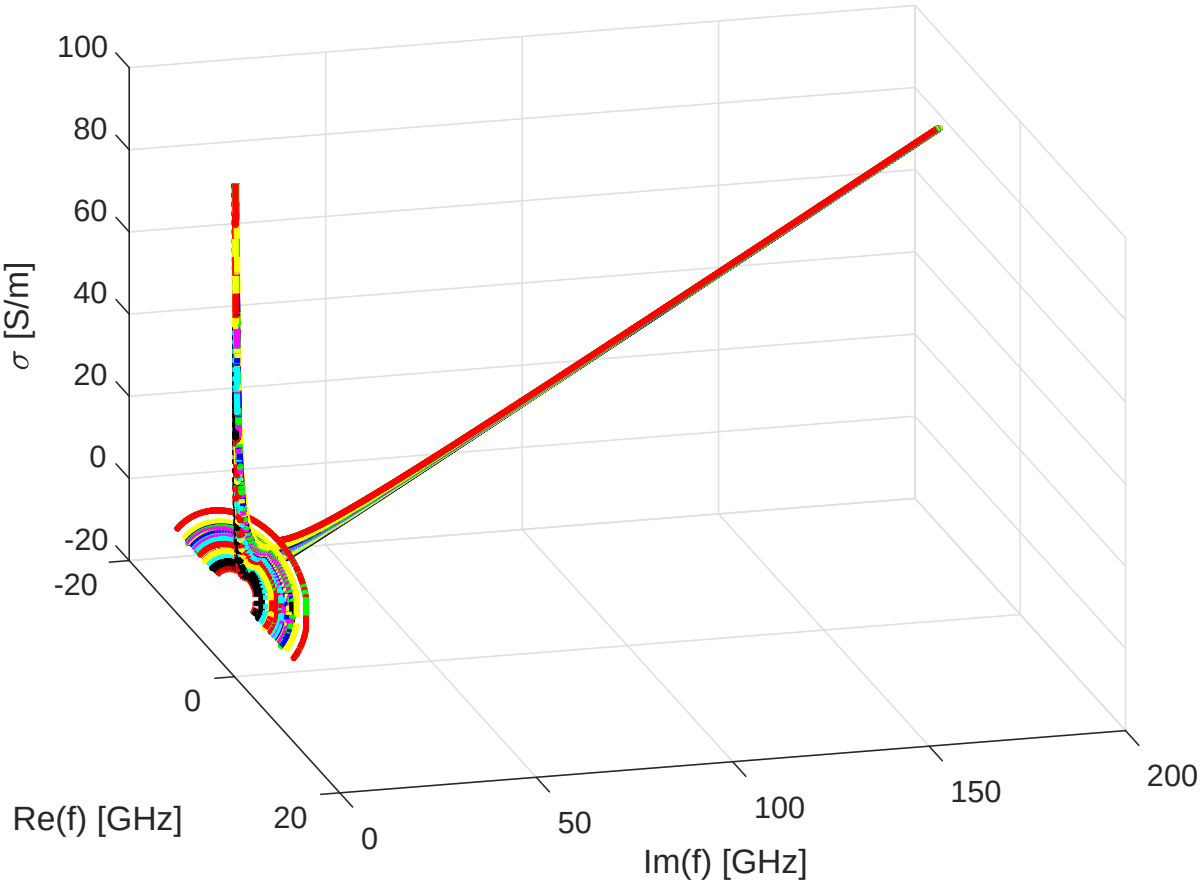

Figure 2: The complex resonance frequencies for a homogeneous cylindrical resonator as a function of conductivity (legend in table 1).

4 Summary

The global root finding algorithm is proven to be very useful in the analysis of the structures filled with lossy material. Finding roots is very efficient. Results of the analysis were confirmed by checking the transition between $\varepsilon_{r2} = 10$ and $\varepsilon_{r2} = 1$ and between lossy material (high value of σ) and perfect electric conductor.

References

[1] P. Kowalczyk, "Complex Root Finding Algorithm Based on Delaunay Triangulation", *ACM Trans. Math. Softw.*, vol. 41,no. 3, pp. 19:1-19:13, Jun. 2015.

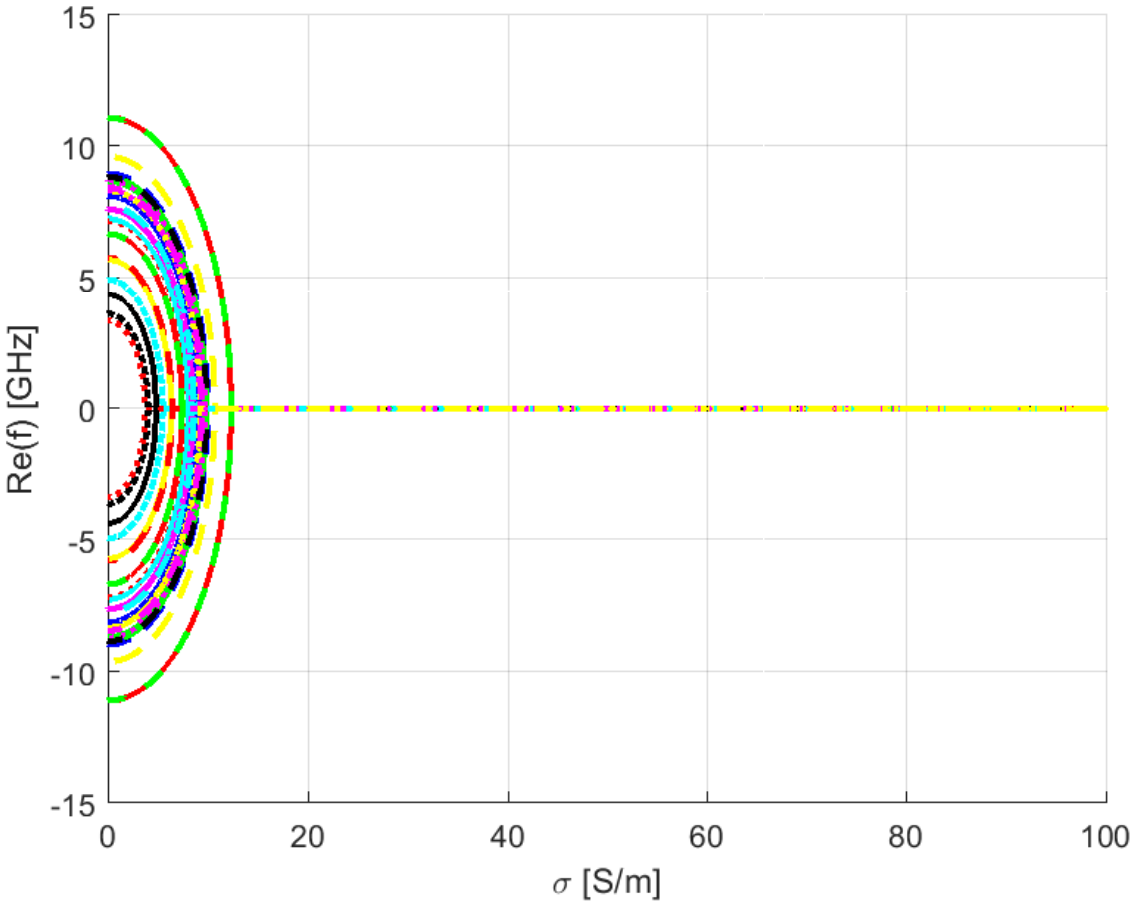

Figure 3: The real part of the resonance frequencies for a homogeneous cylindrical resonator as a function of conductivity (legend in table 1).

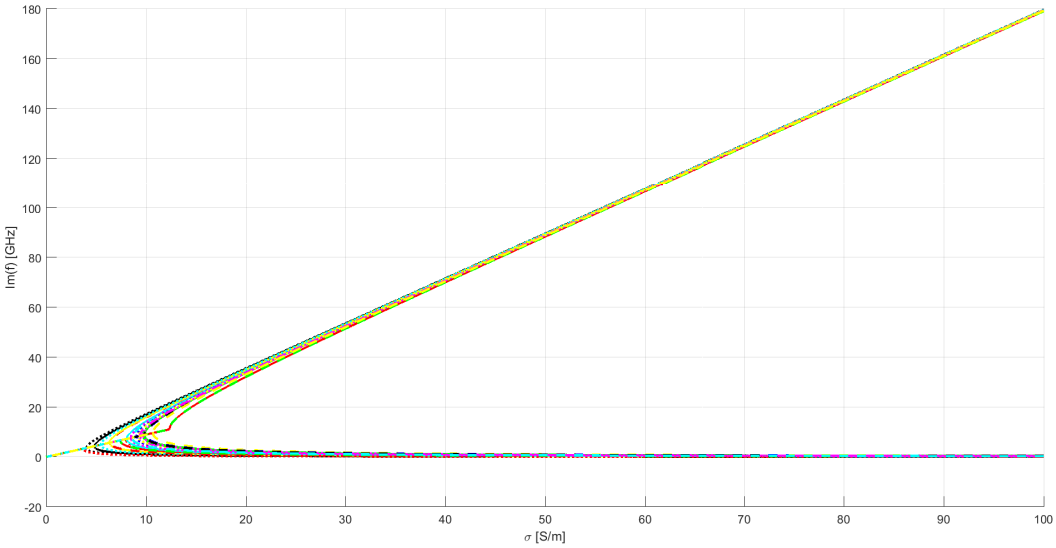

Figure 4: The imaginary part of the resonance frequencies for a homogeneous cylindrical resonator as a function of losses (legend in table 1).

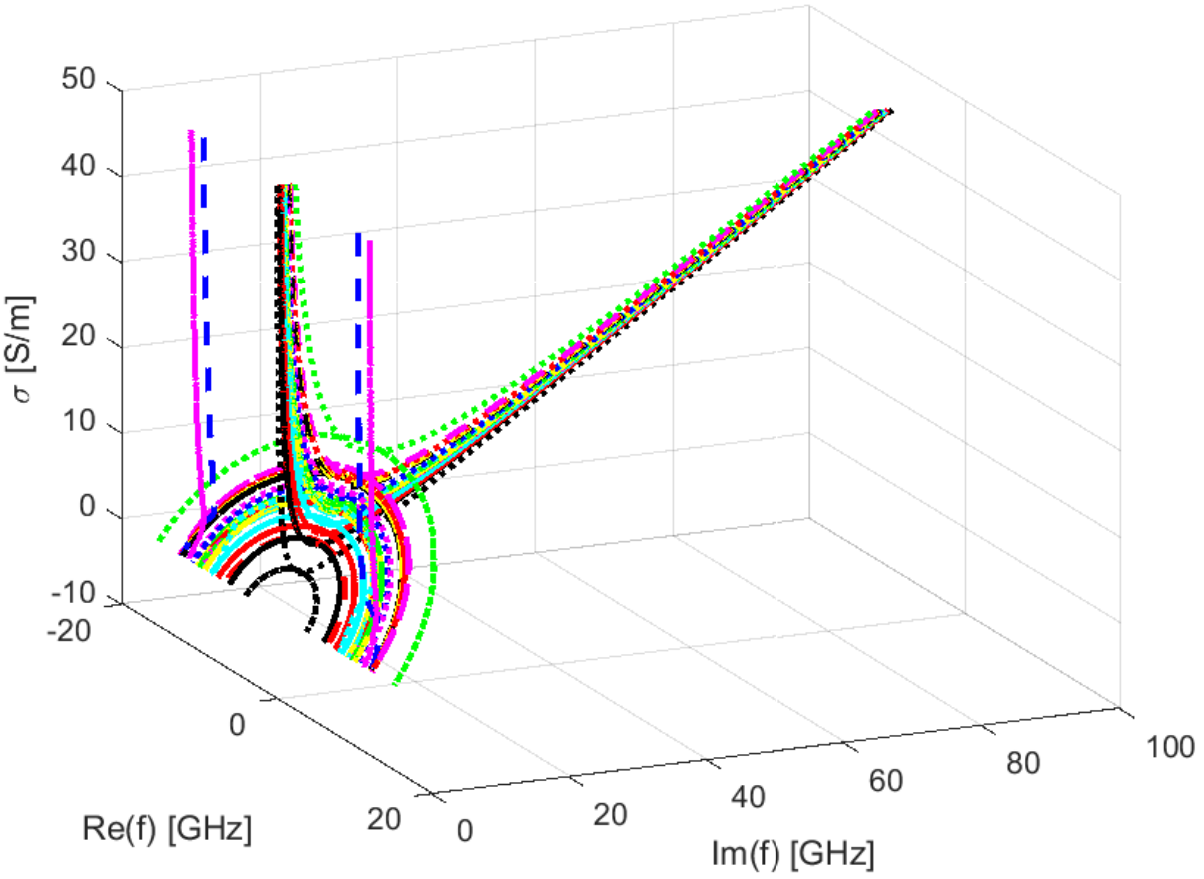

Figure 5: The complex resonance frequencies for a coaxially loaded cylindrical resonator as a function of losses (A legend in the table 5).

Table 6: Modes presented in the Fig.5-7.

mode	color in a figure	line style
HE_{011}	red	solid
HE_{012}	green	solid
HE_{111}	black	solid
HE_{112}	cyan	solid
HE_{121}	magenta	solid
HE_{211}	yellow	solid
HE_{212}	blue	dotted
HE_{311}	red	dotted
HE_{411}	green	dotted
TM_{010}	black	dotted
TM_{011}	cyan	dotted
TM_{012}	magenta	dotted
TM_{020}	yellow	dotted
TM_{021}	blue	dashed
TM_{110}	red	dashed
EH_{111}	green	dashed
EH_{112}	black	dashed
TM_{210}	cyan	dashed
EH_{211}	magenta	dashed
TM_{310}	yellow	dashed

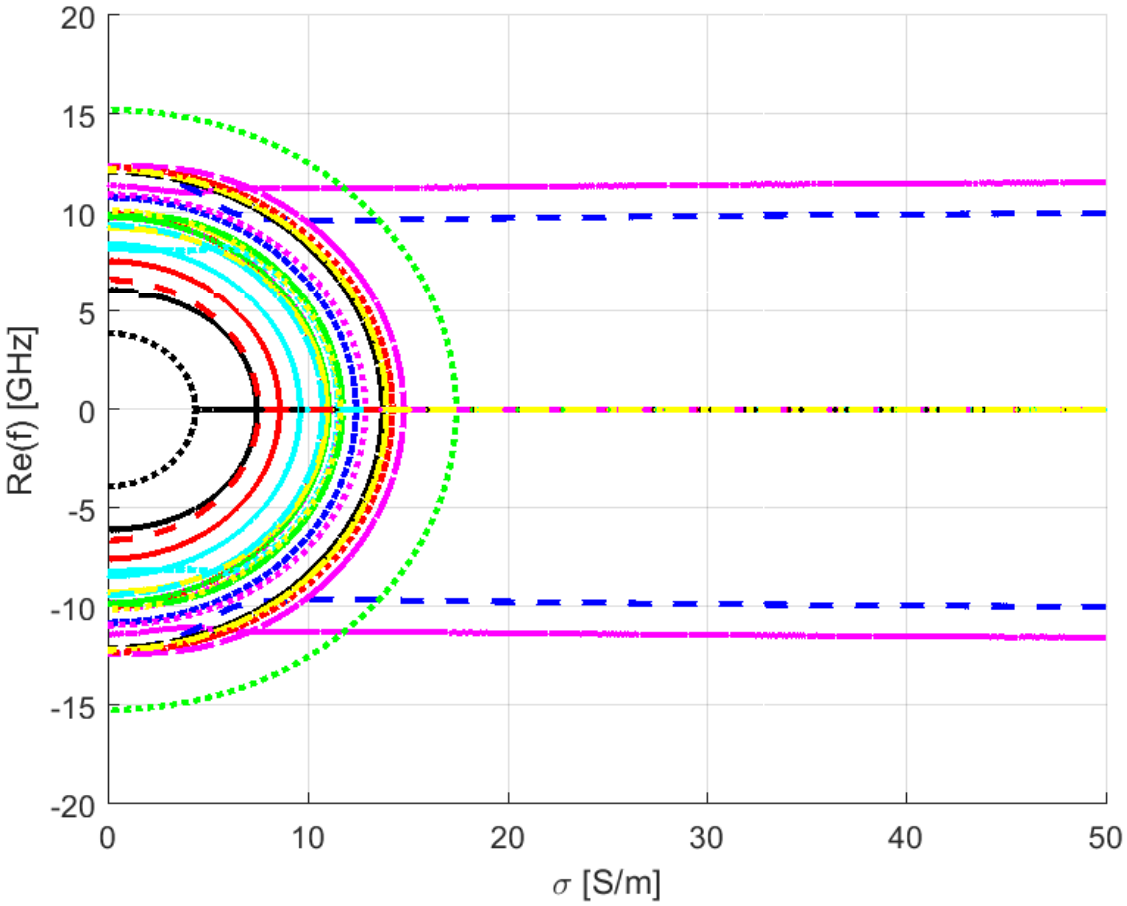

Figure 6: The real part of the resonance frequencies for a coaxially loaded cylindrical resonator as a function of conductivity (legend in table 5).

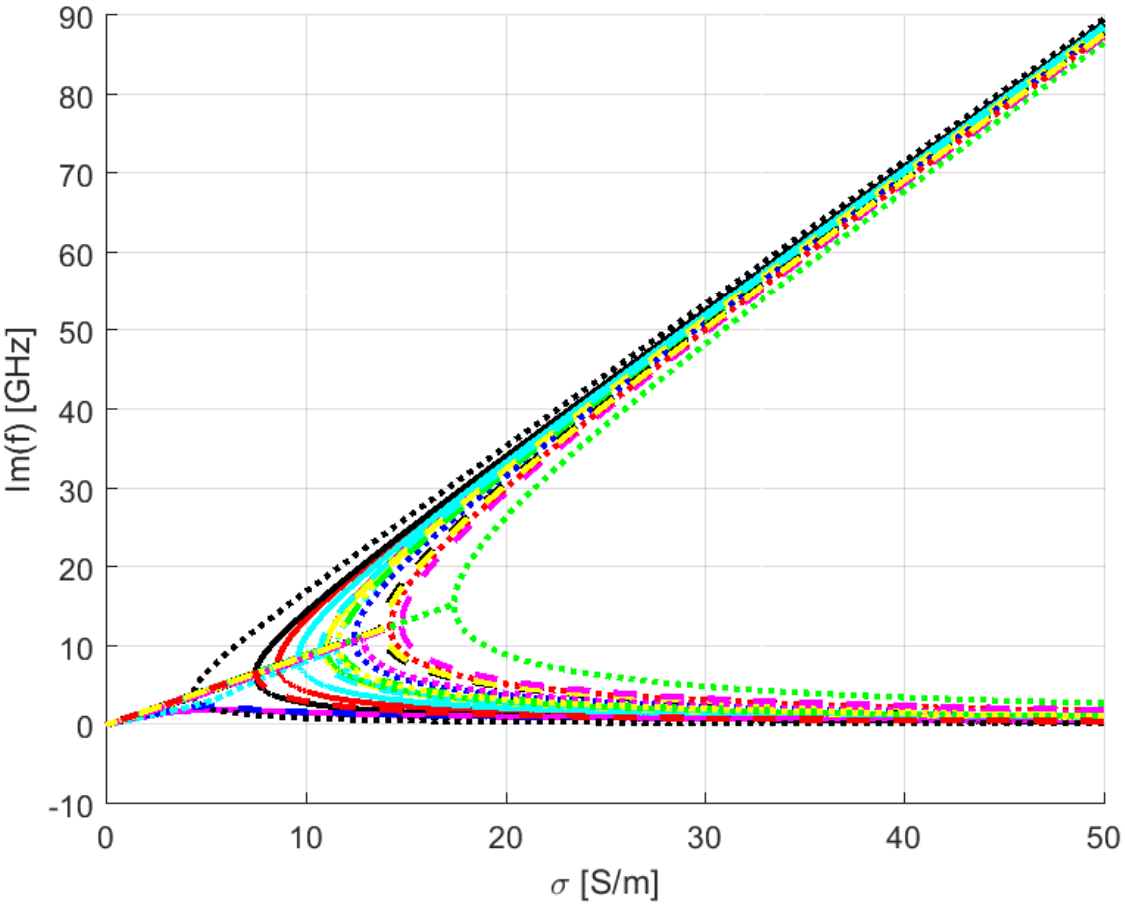

Figure 7: The imaginary part of the resonance frequencies for a coaxially loaded cylindrical resonator as a function of conductivity (legend in table 5).

Table 7: The resonance frequencies of the coaxially loaded resonator as a function of conductivity.

σ	0.01	0.05
TM_{011}	8.193371311 + 0.005620955i	8.193334914 + 0.028104559i
TM_{011}	8.193371311 + 0.005620955i	8.193334914 + 0.028104559i
TM_{021}	12.187611798 + 0.005710941i	12.187494894 + 0.028554449i
TM_{010}	3.883846836 + 0.008860728i	3.883604478 + 0.044303647i
TM_{010}	3.883846836 + 0.008860728i	3.883604478 + 0.044303647i
TM_{020}	10.120289968 + 0.008680030i	10.120201536 + 0.043400162i
TM_{020}	10.120289968 + 0.008680030i	10.120201536 + 0.043400162i
TM_{110}	6.618760142 + 0.008760717i	6.618621574 + 0.043803594i
TM_{110}	6.618760142 + 0.008760717i	6.618621574 + 0.043803594i
TM_{210}	9.389882430 + 0.008708655i	9.389786361 + 0.043543286i
TM_{210}	9.389882430 + 0.008708655i	9.389786361 + 0.043543286i
TM_{310}	12.204093597 + 0.008698429i	12.204020158 + 0.043492153i
TM_{310}	12.204093597 + 0.008698429i	12.204020158 + 0.043492153i
EH_{111}	9.884742887 + 0.007439087i	9.884725417 + 0.037195934i
EH_{111}	9.884742887 + 0.007439087i	9.884725417 + 0.037195934i
EH_{211}	12.403655641 + 0.007566035i	12.403630797 + 0.037830449i
EH_{211}	12.403655641 + 0.007566035i	12.403630797 + 0.037830449i
EH_{112}	12.101983726 + 0.008754416i	12.101909095 + 0.043772094i
EH_{112}	12.101983726 + 0.008754416i	12.101909095 + 0.043772094i
HE_{011}	7.575021632 + 0.008797067i	7.574899551 + 0.043985342i
HE_{011}	7.575021632 + 0.008797067i	7.574899551 + 0.043985342i
HE_{012}	9.809556454 + 0.008865483i	9.809460643 + 0.044327420i
HE_{012}	9.809556454 + 0.008865483i	9.809460643 + 0.044327420i
TM_{012}	10.941787080 + 0.008321628i	10.941715211 + 0.041608179i
TM_{012}	10.941787080 + 0.008321628i	10.941715211 + 0.041608179i
HE_{111}	6.088970461 + 0.007932438i	6.088853781 + 0.039662276i
HE_{111}	6.088970461 + 0.007932438i	6.088853781 + 0.039662276i
HE_{121}	11.393801008 + 0.005813773i	11.393707068 + 0.029067930i
HE_{211}	9.258477841 + 0.008125147i	9.258404003 + 0.040625872i
HE_{211}	9.258477841 + 0.008125147i	9.258404003 + 0.040625872i
HE_{112}	8.439291092 + 0.008753105i	8.439182696 + 0.043765533i
HE_{112}	8.439291092 + 0.008753105i	8.439182696 + 0.043765533i
HE_{311}	12.308459456 + 0.008457335i	12.308395401 + 0.042286733i
HE_{311}	12.308459456 + 0.008457335i	12.308395401 + 0.042286733i
HE_{212}	10.796262626 + 0.008632330i	10.796181356 + 0.043161666i
HE_{212}	10.796262626 + 0.008632330i	10.796181356 + 0.043161666i
HE_{411}	15.220917038 + 0.008627365i	15.220860786 + 0.043136845i
HE_{411}	15.220917038 + 0.008627365i	15.220860786 + 0.043136845i

σ	0.1	0.5
TM_{011}	8.193221216 + 0.056207766i	8.189615971 + 0.280824146i
TM_{011}	8.193221216 + 0.056207766i	8.189615971 + 0.280824146i
TM_{021}	12.187129551 + 0.057107300i	12.175423561 + 0.285279855i
TM_{010}	3.882847010 + 0.088607329i	3.858528887 + 0.443041996i
TM_{010}	3.882847010 + 0.088607329i	3.858528887 + 0.443041996i
TM_{020}	10.119925179 + 0.086800376i	10.111077389 + 0.434010120i
TM_{020}	10.119925179 + 0.086800376i	10.111077389 + 0.434010120i
TM_{110}	6.618188531 + 0.087607238i	6.604315581 + 0.438044227i
TM_{110}	6.618188531 + 0.087607238i	6.604315581 + 0.438044227i
TM_{210}	9.389486139 + 0.087086625i	9.379873518 + 0.435441592i
TM_{210}	9.389486139 + 0.087086625i	9.379873518 + 0.435441592i
TM_{310}	12.203790657 + 0.086984354i	12.196444018 + 0.434929288i
TM_{310}	12.203790657 + 0.086984354i	12.196444018 + 0.434929288i
EH_{111}	9.884670823 + 0.074394986i	9.882923195 + 0.372480214i
EH_{111}	9.884670823 + 0.074394986i	9.882923195 + 0.372480214i
EH_{211}	12.403553149 + 0.075662586i	12.401059710 + 0.378583721i
EH_{211}	12.403553149 + 0.075662586i	12.401059710 + 0.378583721i
EH_{112}	12.101675867 + 0.087544268i	12.094209449 + 0.437734208i
EH_{112}	12.101675867 + 0.087544268i	12.094209449 + 0.437734208i
HE_{011}	7.574518033 + 0.087970723i	7.562298922 + 0.439859883i
HE_{011}	7.574518033 + 0.087970723i	7.562298922 + 0.439859883i
HE_{012}	9.809161225 + 0.088654862i	9.799574879 + 0.443277548i
HE_{012}	9.809161225 + 0.088654862i	9.799574879 + 0.443277548i
TM_{012}	10.941490617 + 0.083216579i	10.934299533 + 0.416118267i
TM_{012}	10.941490617 + 0.083216579i	10.934299533 + 0.416118267i
HE_{111}	6.088489138 + 0.079325075i	6.076806431 + 0.396709619i
HE_{111}	6.088489138 + 0.079325075i	6.076806431 + 0.396709619i
HE_{121}	11.393413534 + 0.058129991i	11.384044884 + 0.289704562i
HE_{211}	9.258173247 + 0.081252596i	9.250776906 + 0.406398800i
HE_{211}	9.258173247 + 0.081252596i	9.250776906 + 0.406398800i
HE_{112}	8.438843948 + 0.087531104i	8.427996479 + 0.437661516i
HE_{112}	8.438843948 + 0.087531104i	8.427996479 + 0.437661516i
HE_{311}	12.308195224 + 0.084573822i	12.301784167 + 0.422925853i
HE_{311}	12.308195224 + 0.084573822i	12.301784167 + 0.422925853i
HE_{212}	10.795927385 + 0.086323420i	10.787796433 + 0.431631229i
HE_{212}	10.795927385 + 0.086323420i	10.787796433 + 0.431631229i
HE_{411}	15.220684998 + 0.086273812i	15.215057695 + 0.431388471i
HE_{411}	15.220684998 + 0.086273812i	15.215057695 + 0.431388471i

σ	1	5
TM_{011}	8.178765458 + 0.560345563i	8.184165712 + 2.848508919i
TM_{011}	8.178765458 + 0.560345563i	8.184165712 + 2.848508919i
TM_{021}	12.138652932 + 0.568935348i	10.767934432 + 2.425160251i
TM_{010}	3.781518960 + 0.886117318i	6.573751903i
TM_{010}	3.781518960 + 0.886117318i	2.297249665i
TM_{020}	10.083373120 + 0.868071630i	9.148529762 + 4.348229816i
TM_{020}	10.083373120 + 0.868071630i	9.148529762 + 4.348229816i
TM_{110}	6.560766410 + 0.876138529i	4.962572954 + 4.388177543i
TM_{110}	6.560766410 + 0.876138529i	4.962572954 + 4.388177543i
TM_{210}	9.349764862 + 0.870935955i	8.324340318 + 4.362636784i
TM_{210}	9.349764862 + 0.870935955i	8.324340318 + 4.362636784i
TM_{310}	12.173453232 + 0.869905450i	11.410108443 + 4.356642312i
TM_{310}	12.173453232 + 0.869905450i	11.410108443 + 4.356642312i
EH_{111}	9.877412788 + 0.748273488i	9.265427451 + 4.185444420i
EH_{111}	9.877412788 + 0.748273488i	9.265427451 + 4.185444420i
EH_{211}	12.393151818 + 0.758879422i	11.977818451 + 4.047516883i
EH_{211}	12.393151818 + 0.758879422i	11.977818451 + 4.047516883i
EH_{112}	12.070837710 + 0.875548084i	11.290976214 + 4.388175965i
EH_{112}	12.070837710 + 0.875548084i	11.290976214 + 4.388175965i
HE_{011}	7.523981348 + 0.879758825i	6.168967803 + 4.404652172i
HE_{011}	7.523981348 + 0.879758825i	6.168967803 + 4.404652172i
HE_{012}	9.769554869 + 0.886575287i	8.752996788 + 4.435934599i
HE_{012}	9.769554869 + 0.886575287i	8.752996788 + 4.435934599i
TM_{012}	10.911776377 + 0.832457045i	10.147887577 + 4.195155460i
TM_{012}	10.911776377 + 0.832457045i	10.147887577 + 4.195155460i
HE_{111}	6.040116668 + 0.793958918i	4.634133366 + 4.078281304i
HE_{111}	6.040116668 + 0.793958918i	4.634133366 + 4.078281304i
HE_{121}	11.355116244 + 0.573343701i	11.098814421 + 1.956280269i
HE_{211}	9.227511901 + 0.813638733i	8.367157908 + 4.172468014i
HE_{211}	9.227511901 + 0.813638733i	8.367157908 + 4.172468014i
HE_{112}	8.394003497 + 0.875360400i	7.218491308 + 4.382427531i
HE_{112}	8.394003497 + 0.875360400i	7.218491308 + 4.382427531i
HE_{311}	12.281682782 + 0.846202112i	11.590631439 + 4.273896444i
HE_{311}	12.281682782 + 0.846202112i	11.590631439 + 4.273896444i
HE_{212}	10.762338792 + 0.863350408i	9.906220907 + 4.329605914i
HE_{212}	10.762338792 + 0.863350408i	9.906220907 + 4.329605914i
HE_{411}	15.197446386 + 0.862897257i	14.614092918 + 4.330563863i
HE_{411}	15.197446386 + 0.862897257i	14.614092918 + 4.330563863i

σ	10	50
TM_{011}	5.013125671 + 8.082320397i	1.062711507i
TM_{011}	5.013125671 + 8.082320397i	88.094642316i
TM_{021}	9.658158439 + 1.632617845i	10.001361011 + 0.657079908i
TM_{010}	16.891374321i	89.564677852i
TM_{010}	0.8963027467i	0.1705263881i
TM_{020}	5.168290966 + 8.737693702i	1.193490361i
TM_{020}	5.168290966 + 8.737693702i	88.204459867i
TM_{110}	3.015641971i	0.502423287i
TM_{110}	14.609775671i	88.328269708i
TM_{210}	3.446264103 + 8.765105550i	88.433098749i
TM_{210}	3.446264103 + 8.765105550i	1.022317464i
TM_{310}	8.554474315 + 8.749768882i	87.621308430i
TM_{310}	8.554474315 + 8.749768882i	1.741975509i
EH_{111}	5.416164437 + 8.681022277i	88.198129721i
EH_{111}	5.416164437 + 8.681022277i	1.211258954i
EH_{211}	9.533642567 + 8.538014300i	1.929323611i
EH_{211}	9.533642567 + 8.538014300i	87.286222675i
EH_{112}	8.340528937 + 8.814170236i	87.805392675i
EH_{112}	8.340528937 + 8.814170236i	1.699975331i
HE_{011}	13.363176911i	88.952455487i
HE_{011}	4.313059812i	0.656512797i
HE_{012}	4.179020621 + 8.887452320i	1.098108685i
HE_{012}	4.179020621 + 8.887452320i	88.566013094i
TM_{012}	7.034355376 + 8.536163719i	1.446938383i
TM_{012}	7.034355376 + 8.536163719i	88.566013094i
HE_{111}	14.303944181i	88.973277409i
HE_{111}	2.769965631i	0.462997396i
HE_{121}	11.267428118 + 1.567485972i	11.556368348 + 0.632450871i
HE_{211}	3.918216906 + 8.599795786i	88.323651569i
HE_{211}	3.918216906 + 8.599795786i	1.042072260i
HE_{112}	11.19953032i	88.650592994i
HE_{112}	6.387075514i	0.820378611i
HE_{311}	8.886854012 + 8.669948369i	87.509117912i
HE_{311}	8.886854012 + 8.669948369i	1.804362029i
HE_{212}	6.448047153 + 8.717360493i	87.992859637i
HE_{212}	6.448047153 + 8.717360493i	1.366178046i
HE_{411}	12.548575679 + 8.720663650i	2.752193243i
HE_{411}	12.548575679 + 8.720663650i	86.522940987i

Table 8: The Q factor of the coaxially loaded resonator as a function of conductivity.

σ	0.01	0.05	0.1	0.5	1	5	10	50
TM_{011}	728.824	145.765	72.883	14.581	7.298	1.437	0.310	0.000
TM_{011}	728.824	145.765	72.883	14.581	7.298	1.437	0.310	0.000
TM_{021}	1067.041	213.408	106.704	21.339	10.668	2.220	2.958	7.610
TM_{010}	219.161	43.829	21.910	4.355	2.134	0.000	0.000	0.000
TM_{010}	219.161	43.829	21.910	4.355	2.134	0.000	0.000	0.000
TM_{020}	582.964	116.592	58.294	11.648	5.808	1.052	0.296	0.000
TM_{020}	582.964	116.592	58.294	11.648	5.808	1.052	0.296	0.000
TM_{110}	377.752	75.549	37.772	7.538	3.744	0.565	0.000	0.000
TM_{110}	377.752	75.549	37.772	7.538	3.744	0.565	0.000	0.000
TM_{210}	539.112	107.821	53.909	10.771	5.368	0.954	0.197	0.000
TM_{210}	539.112	107.821	53.909	10.771	5.368	0.954	0.197	0.000
TM_{310}	701.511	140.301	70.149	14.021	6.997	1.310	0.489	0.000
TM_{310}	701.511	140.301	70.149	14.021	6.997	1.310	0.489	0.000
EH_{111}	664.379	132.874	66.434	13.266	6.600	1.107	0.312	0.000
EH_{111}	664.379	132.874	66.434	13.266	6.600	1.107	0.312	0.000
EH_{211}	819.693	163.937	81.966	16.378	8.165	1.480	0.558	0.000
EH_{211}	819.693	163.937	81.966	16.378	8.165	1.480	0.558	0.000
EH_{112}	691.193	138.238	69.117	13.815	6.893	1.287	0.473	0.000
EH_{112}	691.193	138.238	69.117	13.815	6.893	1.287	0.473	0.000
HE_{011}	430.542	86.107	43.051	8.596	4.276	0.700	0.000	0.000
HE_{011}	430.542	86.107	43.051	8.596	4.276	0.700	0.000	0.000
HE_{012}	553.244	110.648	55.322	11.054	5.510	0.987	0.235	0.000
HE_{012}	553.244	110.648	55.322	11.054	5.510	0.987	0.235	0.000
TM_{012}	657.431	131.485	65.741	13.138	6.554	1.209	0.412	0.000
TM_{012}	657.431	131.485	65.741	13.138	6.554	1.209	0.412	0.000
HE_{111}	383.802	76.759	38.377	7.659	3.804	0.568	0.000	0.000
HE_{111}	383.802	76.759	38.377	7.659	3.804	0.568	0.000	0.000
HE_{121}	979.897	195.984	97.999	19.648	9.903	2.837	3.594	9.136
HE_{211}	569.742	113.947	56.972	11.381	5.671	1.003	0.228	0.000
HE_{211}	569.742	113.947	56.972	11.381	5.671	1.003	0.228	0.000
HE_{112}	482.074	96.414	48.205	9.628	4.795	0.824	0.000	0.000
HE_{112}	482.074	96.414	48.205	9.628	4.795	0.824	0.000	0.000
HE_{311}	727.680	145.535	72.766	14.544	7.257	1.356	0.513	0.000
HE_{311}	727.680	145.535	72.766	14.544	7.257	1.356	0.513	0.000
HE_{212}	625.339	125.067	62.532	12.497	6.233	1.144	0.370	0.000
HE_{212}	625.339	125.067	62.532	12.497	6.233	1.144	0.370	0.000
HE_{411}	882.130	176.425	88.212	17.635	8.806	1.687	0.719	0.000
HE_{411}	882.130	176.425	88.212	17.635	8.806	1.687	0.719	0.000

Table 9: The resonance frequencies of the coaxially loaded with perfect electric conductor resonator .

mode	TM_{021}	HE_{121}
$\sigma = 50$	10.001361011 + 0.657079908i	11.556368348 + 0.632450871i
$\sigma = 100$	10.163284506 + 0.455216508i	11.707184913 + 0.436944723i
$\sigma = 500$	10.399584353 + 0.200637890i	11.931322113 + 0.192012576i
$\sigma \to \infty$	10.599264000	12.122082950
mode	TEM_1	TE_{111}