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flexIble SensOrs

FNP Team-Tech

Reliable greedy multi-point model order reduction for frequency-dependent bi-static radar cross-section

dr eng. Grzegorz Fotyga

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1 Introduction

The aim of this report is to verify the accuracy of the reliable greedy multi-point model order reduction (RGM-MOR) approach for speeding up the simulations of electromagnetic field scattered from arbitrary metallic object. Namely, we consider the frequency-dependent bi-static radar cross-section kind of scattering simulations.

2 Standard scattering formulation

The subsequent steps of the standard scattering formulation approach, (without model-order reduction) are listed below:

1. The geometry of the structure to be analyzed is defined in InventSIM. It is important to enclose the structure in the box, surfaces of which are placed 'far enough' (formulation?) from the object. On the box the absorbing boundary conditions (ABC) of the first kind are applied.

2. The plane wave excitation is applied. It is based on the formulation [1], in which the currents M_s and J_S are impressed o the metallic surfaces of the analysed object. The following parameters of a plane wave have to be specified:

   - Unitary wave propagation vector $\hat{k}$ (e.g. $[1,0,0],[0,0,1]$ etc.)
   - Electric field polarization $\vec{E}_0$. It has to be orthonormal to $\hat{k}$!
   - Frequency, which is a vector $f = \{f_1, f_2 \dots f_n\}$ and n is a number of frequency points in the bandwidth.

3. In subsequent step the system of equation is constructed in which the left-hand side is associated with the geometry and materials of the analysed structures, while the right hand side deals with the excitation (M_s and J_S currents):

$$(\mathbf{\Gamma} + s\mathbf{G} + s^2\mathbf{C})\mathbf{e}(s) = s\mathbf{b}(s), \tag{1}$$

   where $\mathbf{\Gamma}, \mathbf{G}, \mathbf{C} \in \mathbb{C}^{N \times N}$ are system matrices, $s = j\omega/c$, $\mathbf{b}(s) \in \mathbb{C}^N$ is the excitation vector and $\mathbf{e}(s) \in \mathbb{C}^N$ is a matrix of unknown FE coefficients. It should be emphasised that the $\mathbf{b}(s)$ vector exhibits a non-affine frequency dependence.

4. The system of equations (1) is solved, giving the electric field distribution in the near-field zone.

5. Next, the far field is computed using the near-to-far (NTF) field transformation. To this end the integrals of the field on the surfaces of ABC have to be computed. Note that the far-field region defined as: $R > 2D^2/\lambda$, where D is a largest dimension of scatterer and λ is a wavelength. One has to also specify the points in the spherical coordinates system (r, θ, ϕ) in which the far field is to be computed, where $r = R$.

6. The last step is the post-processing, which is usually the plot of the magnitude of the filed in the far-field zone. In this report we concentrate on the simulations which deal with the scattering field computation in the frequency-domain.

3 Scattering formulation enriched with MOR

In this section the scattering field computation procedure enriched with the model order reduction approach. The reduction is applied to the (in most cases) most time consuming step, which is the solution of the system of equation associated with the near-field zone (Section 2, step 4), while the rest of the steps remain unchanged.

The reduced-order model, which approximates the properties of the FEM system (1) can be obtained by means of one of the reliable greedy multipoint model order reduction (RGM-MOR) described in details in [2]:

$$(\mathbf{\Gamma}_R + s\mathbf{G}_R + s^2\mathbf{C}_R)\mathbf{e}_R(s) = s\mathbf{b}_R(s), \tag{2}$$

where $\mathbf{\Gamma}_R = Q^T\mathbf{\Gamma}Q$, $\mathbf{G}_R = Q^T\mathbf{G}Q$, $\mathbf{C}_R = Q^T\mathbf{C}Q$, $\mathbf{b}_R(s) = Q^T\mathbf{b}_R(s)$ are reduced system matrices, $Q \in \mathbb{C}^{N\times q}$ is the orthonormal projection matrix, and q denotes the reduced order, where $q \ll N$. The reduced-order model is created in an automated and computationally efficient way and it is accurate over a wide frequency band. The accuracy and efficiency of the RGM-MOR approach is ensured by the efficiently computed *Z-param goal-oriented error estimator*, which is well-correlated with the real error. The details of the error estimator formula are provided in [3].

What is important, since the right-hand side vector $\mathbf{b}(s)$ exhibits a non-affine frequency dependence, the reduced order model as well as the error estimator should be computed using the formulations provided in EDISON Report 1 [4]. More precisely, the $\mathbf{b}(s)$ vector is subject to proper orthogonal decomposition (POD) which generates the basis of a few vectors that span the whole subspace of excitation vectors $\mathbf{b}(s)$. In effect, one can express $\mathbf{b}(s)$ as a linear combination of few (M) vectors, since the field pattern of the plane wave is in general not strongly-dispersive:

$$\mathbf{b}(s) \approx \sum_{j=1}^{M} a_j(s) \cdot \mathbf{b}_j. \tag{3}$$

Note, that in order to obtain coefficients $a_j(s)$ and vectors $\mathbf{b}_j$ one has to perform the Proper Orthogonal Decomposition (POD) or Reduced Basis Method (RBM) for the whole frequency band. Let us also denote $\mathbf{b}_R = \{\mathbf{b}_1\ \mathbf{b}_2 \ldots \mathbf{b}_N\}$ and $\mathbf{a}_R(s) = \{a_1(s)\ a_2(s) \ldots a_N(s)\}$:

$$\mathbf{b}(s) \approx \mathbf{b}_R \cdot \mathbf{a}_R(s). \tag{4}$$

In effect, the frequency dependence is pushed to the scalar functions $a(s)$ and the equation-to-be-reduced exhibits affine nature. The subsequent steps of the RGM-MOR approach are performed for the right hand side approximated by these few vectors. Eq. (4) is used to approximate RHS in the SAPOR procedure, in solution of (2) and in error estimator:

$$\begin{aligned}
E_{fast}(s) = max\{\cdot\mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{r}(s))\\
/|2s \cdot \mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{b}_R) \cdot \mathbf{a}_R(s)|\} =\\
max\{\cdot(2s\mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{b}_R) \cdot \mathbf{a}_R(s) -\\
\mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{\Gamma} \cdot \mathbf{Q}) \cdot \mathbf{e}_R -\\
s\mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{G} \cdot \mathbf{Q}) \cdot \mathbf{e}_R -\\
s\mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot (\mathbf{b}_R \cdot \mathbf{a}_R(s)) \cdot (\mathbf{b}_R \cdot \mathbf{a}_R(s))^T\mathbf{Q}) \cdot \mathbf{e}_R -\\
s^2\mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{C} \cdot \mathbf{Q}) \cdot \mathbf{e}_R)\\
/|2s \cdot \mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{b}_R) \cdot \mathbf{a}_R(s)|\}
\end{aligned} \tag{5}$$

where:

$$\begin{aligned}\mathbf{r}(s) = 2s(\mathbf{b}_R \cdot \mathbf{a}_R(s)) - \\ \mathbf{\Gamma} \cdot \mathbf{Q} \cdot \mathbf{e}_R - \\ s\mathbf{G} \cdot \mathbf{Q} \cdot \mathbf{e}_R - \\ s(\mathbf{b}_R \cdot \mathbf{a}_R(s)) \cdot (\mathbf{b}_R \cdot \mathbf{a}_R(s))^T \cdot \mathbf{Q} \cdot \mathbf{e}_R - \\ s^2\mathbf{C} \cdot \mathbf{Q} \cdot \mathbf{e}_R.\end{aligned} \tag{6}$$

To sum-up: the RGM-MOR approach is a fully automatic black-box reduction procedure, controlled by the reliable and efficient error estimator.

Finally, in order to transform the near field to the far field zone, a full solution vector $\mathbf{e}(s)$ is required (as the input at the step 5). However, the model order reduction provides the approximated solution (up to the specified accuracy), computed at each frequency (s_i) by:

$$\mathbf{e}(s_i) \approx \mathbf{Q}\mathbf{e}_R(s_i). \tag{7}$$

4 Numerical Experiments

The final section contains the results of the numerical experiments performed using the techniques described in section 2 and 3. The two PEC (perfect electric conductor) structures are herein considered:

- A cube - the length of the edge: 10 mm. It is placed centrally in the air-cube with ABC surfaces with the edges of the length 50 mm. Fig. 1 a).

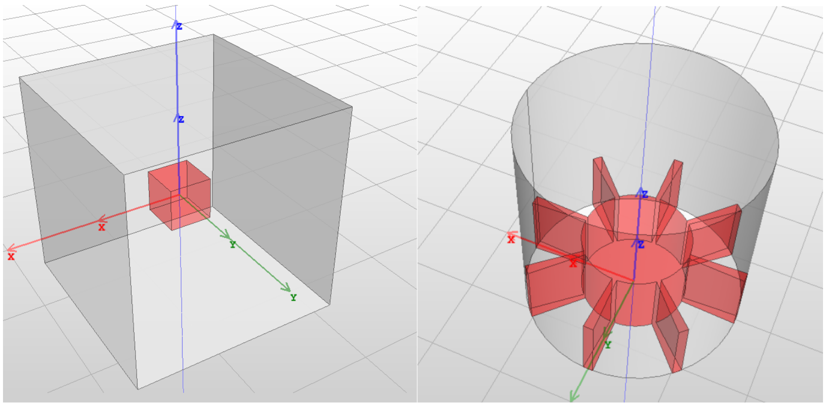

Figure 1: a) PEC (perfect electric conductor) cube, b) Turbina Valentina

- The Inlet Turbine (so called - Turbina Valentina). The dimensions of the structure are provided in [5]. Fig. 1 b).

1. **Experiment 1. PEC cube.**

   The first test deals with very simple structure - PEC cube. The FEM discretization leads to the system of equations with as many as 52066 variables. The plane wave E-field polarization is $\vec{i}_x$, whereas the wave propagation vector is $\hat{k} = \vec{i}_z$. The frequency bandwidth is: 30-35 GHz, with the step 0.1 GHz.

   The RGM-MOR parameters are as follows: maximum value of vectors in the projection basis, in each of the expansion points: $q_{max} = 32$ vectors and the tolerance: $tol = 1e-3$, whereas the accuracy in the POD approach $= 1e-6$. The characteristics of the structure before, after reduction and the real error (the difference of the magnitude of the far-field electric field) are provided in Fig. 2.

   The table 1 contains the computational time of subsequent MOR steps.

Reduction time	11.72s
Initial time (POD)	4.99s
Solution time	5.29s
Orthogonalization	0.89s
Error estimation	0.025s
Matrices update	0.42s
Size of a basis	55
Expansion points	3
Global max error	0.0001
Approx. Speed up	2.1

2. **Experiment 2. Turbina Valentina**

The second test deals with the inlet turbine. The FEM discretization leads to the system of equations with as many as 2342264 variables. The plane wave E-field polarization is $\vec{i}_y$, whereas the wave propagation vector is $\hat{k} = -\vec{i}_z$. The frequency bandwidth is: 1.5-2 GHz, with the step 0.02 GHz.

The RGM-MOR parameters are as follows: maximum value of vectors in the projection basis, in each of the expansion points: $q_{max} = 20$ vectors and the tolerance: $tol = 1e-4$, whereas the accuracy in the POD approach $= 1e-6$. The table 2 contains the computational time of subsequent MOR steps.

Reduction time	2921.2s
Initial time (POD)	293.0s
Solution time	2344.1s
Orthogonalization	200.3s
Error estimation	0.129s
Matrices update	74.7s
Size of a basis	105
Expansion points	4
Global max error (estimated)	0.00007
Approx. Speed up	7.47

Next, we set the reduction parameters to: maximum value of vectors in the projection basis, in each of the expansion points: $q_{max} = 32$ vectors and the tolerance: $tol = 1e-5$ and the accuracy in the POD approach $= 1e-12$. The computational time of the subsequent steps is listed below:

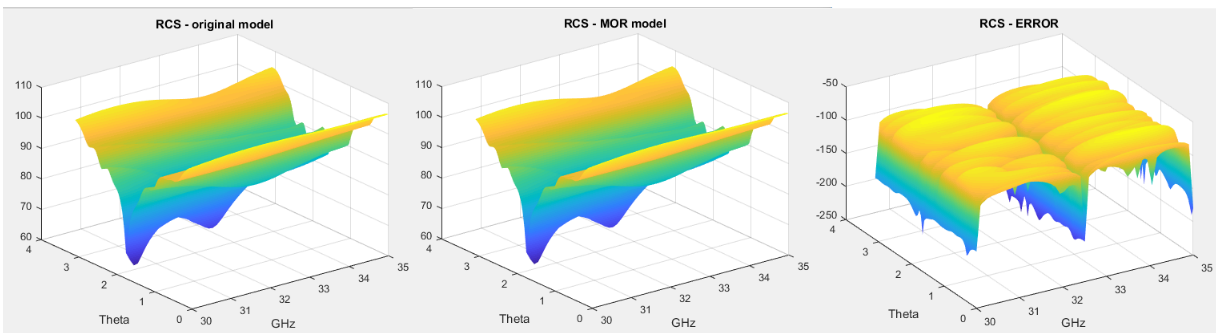

Figure 2: PEC cube

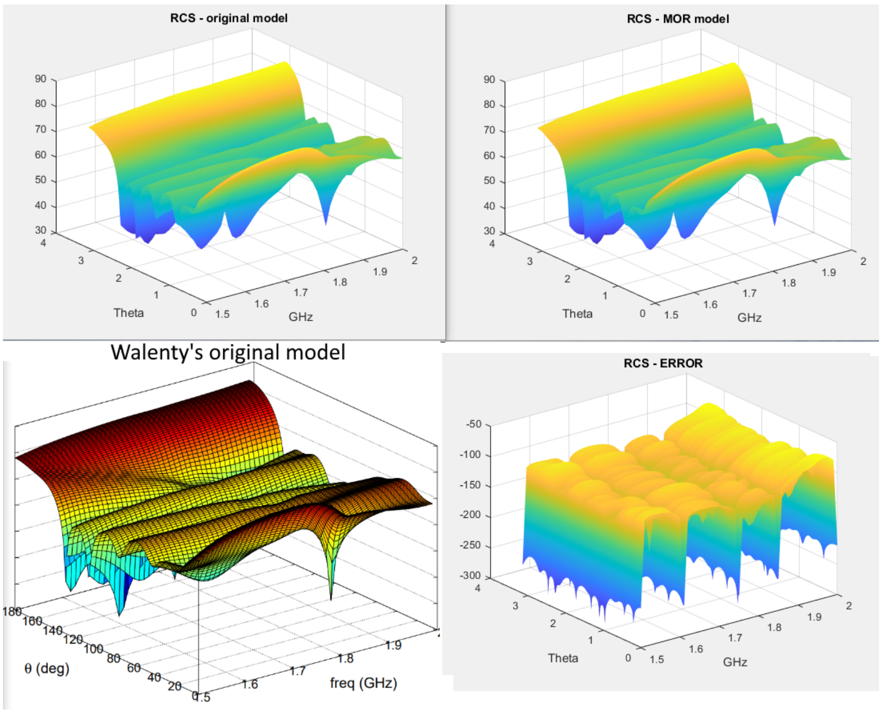

Figure 3: Inlet turbine

Reduction time	3794.8s
Initial time (POD)	306.2s
Solution time	3044.2s
Orthogonalization	322.4s
Error estimation	0.2s
Matrices update	104.3s
Size of a basis	105
Expansion points	5
Global max error (estimated)	0.000009
Approx. Speed up	6.06

The characteristics of the structure before, after reduction and the real error (the difference of the magnitude of the far-field electric field) as well as the reference results from [5] are provided in Fig. 3.

5 Estimator inaccuracy

Although the reduction error is at the low-level, the scattering characteristics are indistinguishable, the real error is not bounded by the estimated one. For example in the last case, the estimated error is below -100dB and the reduction procedure is stopped, while the real error, computed for the far-field is at the level -70dB. The reason is that the field computed by means of the reduced model (applied in the near-field region) is transformed to the far-field region. **The error estimator is computed for the near-field (while constructing the reduced model), whereas the real-error applies to the far field.**

References

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