EDISON

Electromagnetic Design of
flexIble SensOrs

FNP Team-Tech

Raport 1. Multipoint Reduction Approach for Non-Affine Right Hand Side Problems

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1 Introduction

Standard global multipoint model order reduction, enhanced by the error estimator tool [1] for finite element method problems is performed on the system of equations of the following form:

$$(\mathbf{\Gamma} + s\mathbf{G} + s^2\mathbf{C})\mathbf{E}(s) = s\mathbf{B}\mathbf{I}, \tag{1}$$

where:

$$\mathbf{B} = c\widetilde{\mathbf{B}}/(Z_P^{1/2}). \tag{2}$$

The approach [1] requires, that the right hand side of (1) – matrix $\widetilde{\mathbf{B}}$ is constant, while the port impedance (Z_P) depends on s. Therefore, approach [1] can be applied in the analysis of the structures excited through f.e. homogeneous waveguide ports or coaxial lines. In the case of micro-strip lines and inhomogeneous ports one has to derive a novel error-estimator, which takes into account, that the matrix $\widetilde{\mathbf{B}}$ is frequency dependent (non-affine).

The goal of this report is to derive the multipoint reduction formulation for problems, with non-affine RHS.

2 Standard error estimator (for affine RHS)

The goal-oriented error estimator is based on a residual error and is defined as follows:

$$E_s(s) = max_i\{\eta_i \cdot (\mathbf{b}_i)^T \cdot (\mathbf{r}_i(s))/|2s\eta_i(\mathbf{b}_i)^T \cdot \mathbf{b}_i|\}, \tag{3}$$

where $\mathbf{r}_i$ is the i-th residual vector and $\mathbf{b}_i$ is the i-th vector of the matrix $\widetilde{\mathbf{B}}$. See [1] for details, eq. (24) and (25) – for slow and fast computation of an a-posteriori estimator, respectively. If the structure is excited by means of a single-mode, single-port, eq. (3) is simplified to:

$$E_s(s) = max\{\eta \cdot (\mathbf{b})^T \cdot (\mathbf{r}(s))/|2s\eta(\mathbf{b})^T \cdot \mathbf{b}|\}, \tag{4}$$

3 Error estimator (for non-affine RHS)

In the following parts of the rep. we assume, that the structure is excited by means of a single-port and single-mode. If the excitation exhibits non-affine nature (which means that $\mathbf{b}$ depends on s), one can express $\mathbf{b}(s)$ as a linear combination of few (N) vectors, since the field pattern at the ports is in general not strongly-dispersive:

$$\mathbf{b}(s) \approx \sum_{j=1}^{N} a_j(s) \cdot \mathbf{b}_j. \tag{5}$$

Note, that in order to obtain coefficients $a_j(s)$ and vectors $\mathbf{b}_j$ one has to perform the Proper Orthogonal Decomposition (POD) or Reduced Basis Method (RBM) for the whole frequency band. Let us also denote $\mathbf{b}_R = \{\mathbf{b}_1\ \mathbf{b}_2 \ldots \mathbf{b}_N\}$ and $\mathbf{a}_R(s) = \{a_1(s)\ a_2(s) \ldots a_N(s)\}$:

$$\mathbf{b}(s) \approx \mathbf{b}_R \cdot \mathbf{a}_R(s). \tag{6}$$

In effect, the frequency dependence is pushed to the scalar functions $a(s)$ and the equation-to-be-reduced exhibits affine nature.

Substituting (5) to (4) yields:

$$
\begin{aligned}
E(s) = max\{\eta \cdot (\mathbf{b}(\mathbf{s}))^T \cdot (\mathbf{r}(s))/|2s\eta(\mathbf{b}(\mathbf{s}))^T \cdot \mathbf{b}(\mathbf{s})|\} \approx \\
\approx max\{\eta \cdot (\sum_{j=1}^{N} a_j(s) \cdot \mathbf{b}_j)^T \cdot (\mathbf{r}(s)) \\
/|2s\eta(\sum_{j=1}^{N} a_j(s) \cdot \mathbf{b}_j)^T \cdot \sum_{j=1}^{N} a_j(s) \cdot \mathbf{b}_j|\} = \\
= max\{\eta \cdot \mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{r}(s)) \\
/|2s\eta \cdot \mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{b}_R) \cdot \mathbf{a}_R(s)|\}.
\end{aligned} \tag{7}
$$

Finally, the *fast* formula for the error estimator can be derived, taking into account that the residual vector is defined as follows:

$$
\begin{aligned}
\mathbf{r}(s) = 2s\mathbf{b}(s) - \\
\mathbf{\Gamma} \cdot \mathbf{Q} \cdot \mathbf{e}_r - \\
s\mathbf{G} \cdot \mathbf{Q} \cdot \mathbf{e}_r - \\
s\mathbf{b}(s) \cdot \mathbf{b}(s)^T \cdot \mathbf{Q} \cdot \mathbf{e}_r - \\
s^2\mathbf{C} \cdot \mathbf{Q} \cdot \mathbf{e}_r,
\end{aligned} \tag{8}
$$

where $\mathbf{e}_r$ denotes N vectors of unknowns obtained from the reduced model:

$$
(\widehat{\mathbf{\Gamma}} + s\widehat{\mathbf{G}} + s^2\widehat{\mathbf{C}})\mathbf{e}_r(s) = s(\widehat{\mathbf{b}}_R \cdot \mathbf{a}_R(s)) \cdot \mathbf{I}. \tag{9}
$$

Substituting (6) to (8) yields:

$$
\begin{aligned}
\mathbf{r}(s) = 2s(\mathbf{b}_R \cdot \mathbf{a}_R(s)) - \\
\mathbf{\Gamma} \cdot \mathbf{Q} \cdot \mathbf{e}_r - \\
s\mathbf{G} \cdot \mathbf{Q} \cdot \mathbf{e}_r - \\
s(\mathbf{b}_R \cdot \mathbf{a}_R(s)) \cdot (\mathbf{b}_R \cdot \mathbf{a}_R(s))^T \cdot \mathbf{Q} \cdot \mathbf{e}_r - \\
s^2\mathbf{C} \cdot \mathbf{Q} \cdot \mathbf{e}_r.
\end{aligned} \tag{10}
$$

Finally, the fast estimator formula reads:

$$
\begin{aligned}
E_{fast}(s) = max\{\eta \cdot \mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{r}(s)) \\
/|2s\eta \cdot \mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{b}_R) \cdot \mathbf{a}_R(s)|\} = \\
max\{\eta \cdot (2s\mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{b}_R) \cdot \mathbf{a}_R(s) - \\
\mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{\Gamma} \cdot \mathbf{Q}) \cdot \mathbf{e}_r - \\
s\mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{G} \cdot \mathbf{Q}) \cdot \mathbf{e}_r - \\
s\mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot (\mathbf{b}_R \cdot \mathbf{a}_R(s)) \cdot (\mathbf{b}_R \cdot \mathbf{a}_R(s))^T\mathbf{Q}) \cdot \mathbf{e}_r - \\
s^2\mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{C} \cdot \mathbf{Q}) \cdot \mathbf{e}_r) \\
/|2s\eta \cdot \mathbf{a}_R(s)^T \cdot (\mathbf{b}_R^T \cdot \mathbf{b}_R) \cdot \mathbf{a}_R(s)|\}
\end{aligned} \tag{11}
$$

4 Numerical tests

In the numerical test the three structures have been taken into account (Fig. 1).

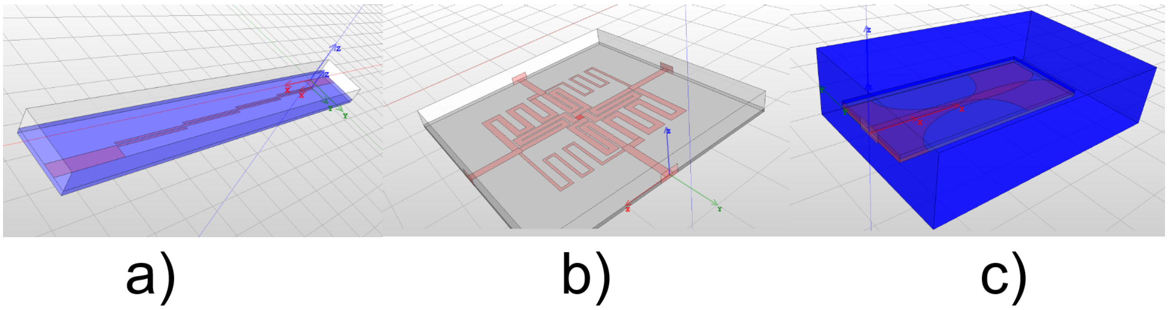

Figure 1: Structures: a) filter, b) coupler, c) Vivaldi antenna.

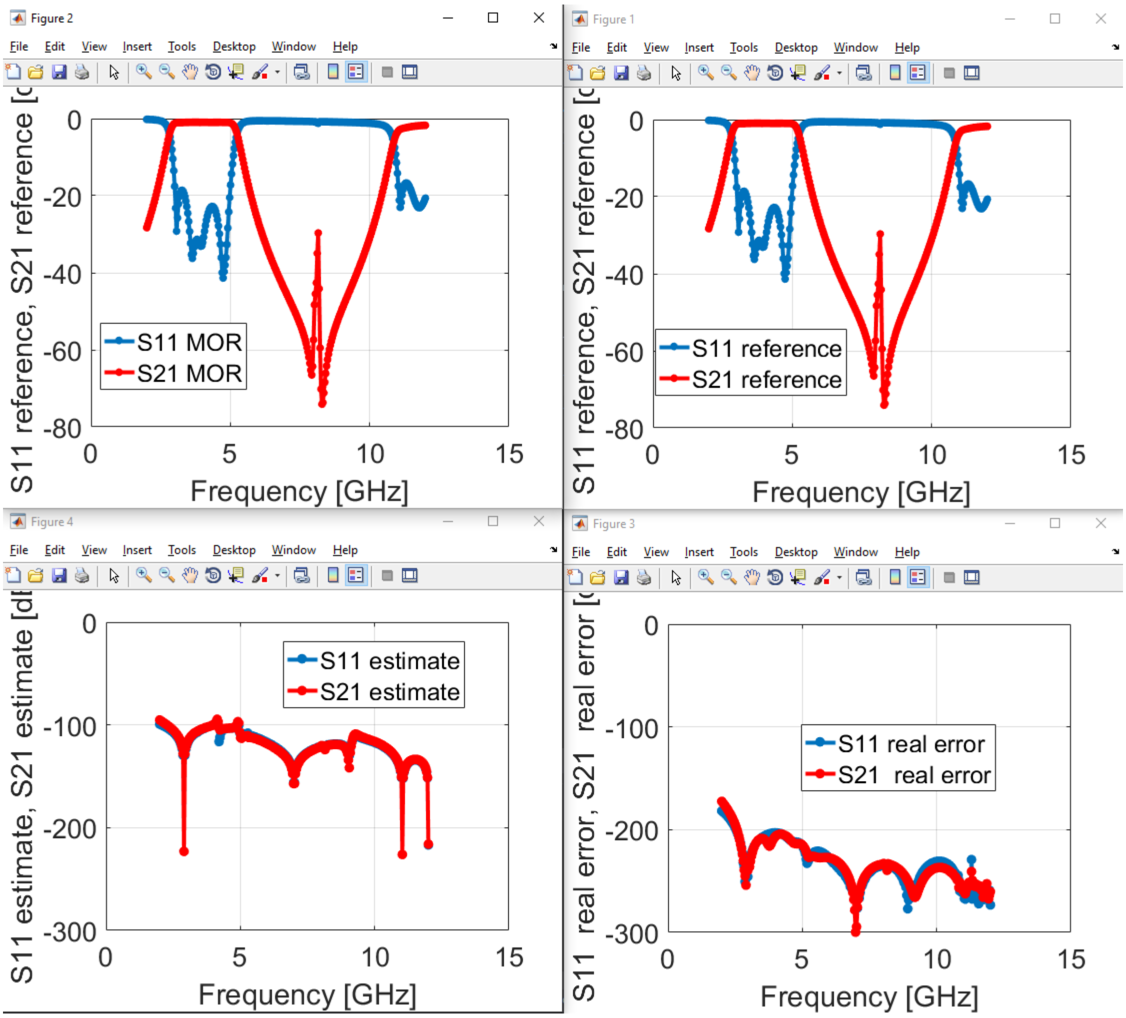

Figure 2: Filter plots.

	Filter	Coupler	Antenna
Number of Frequency point	300	100	101
Frequency bandwidth	2-12 GHz	0.6-2.4 GHz	4-6 GHz
Number of expanding points	4	1	3
Number of vectors in V	54	36	21
Speedup	19.8	10.8	26
Reduction time	83.3 s	23.4 s	108.94 s

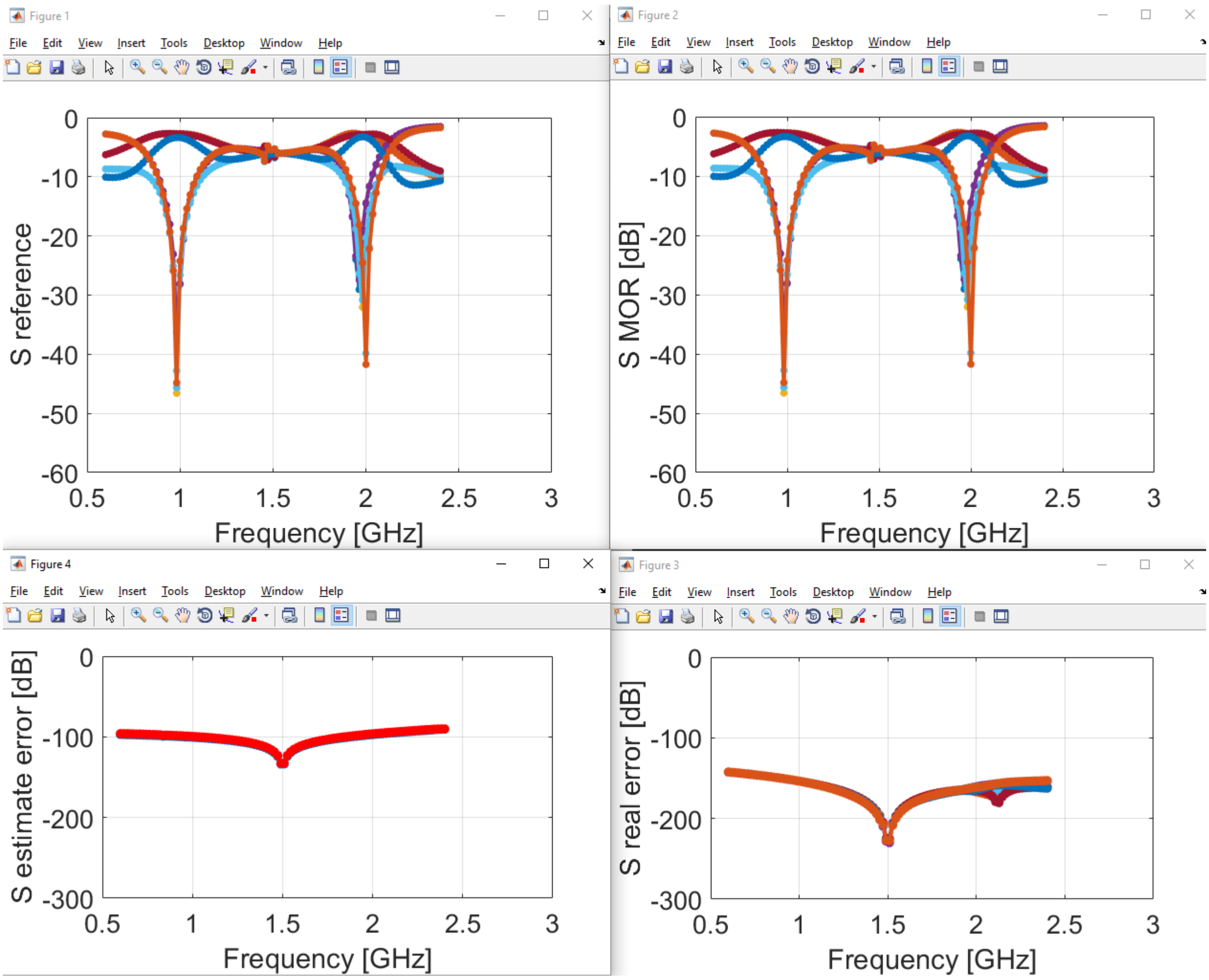

Figure 3: Coupler plots.

References

[1] Rewienski, Michal, Adam Lamecki, and Michal Mrozowski. "Greedy multipoint model-order reduction technique for fast computation of scattering parameters of electromagnetic systems." IEEE Transactions on Microwave Theory and Techniques 64.6 (2016): 1681-1693.

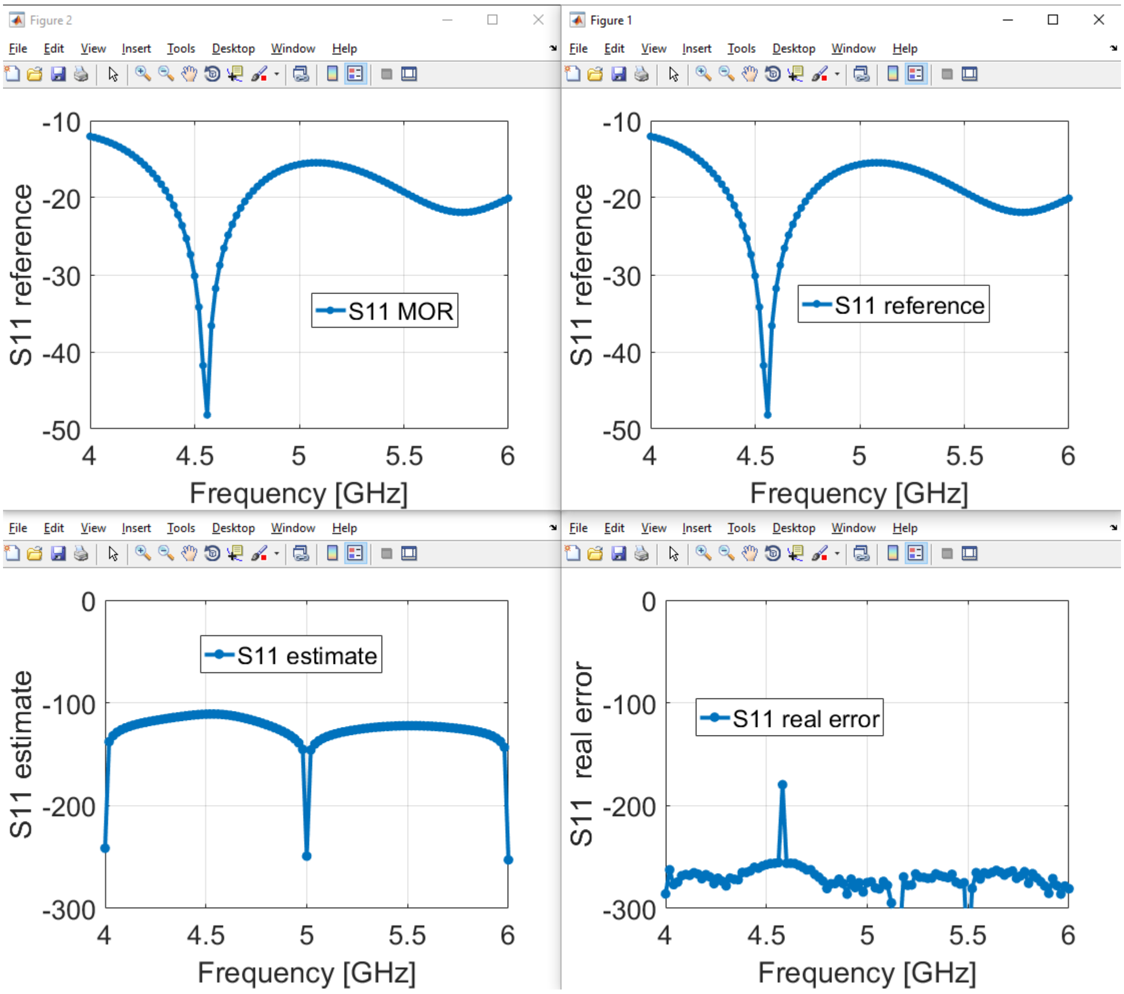

Figure 4: Antenna plots.