



Electromagnetic Design of
flexIble SensOrs



Reliable greedy multi-point model order reduction - performance study

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2.0	22.08.2017		created

1 Introduction

The aim of this report is to verify the accuracy and efficiency of the reliable greedy multi-point model order reduction (RGM-MOR) approach for speeding up the simulations of various microwave structures. The RGM-MOR algorithm is listed in Algorithms 1 and 2, whereas the details of the operations are provided in [1].

Algorithm 1: RGM-MOR - the main loop

Require: $f_{\min}, f_{\max}, tol, q_{\max}, C, G, \Gamma$ and B

- 1: Set: $f = (f_{\min} + f_{\max})/2, \Delta = f_{\max} - f_{\min}, Q_G = [], j = 1, q = 0, N_{\max}^{\text{moments}}$
- 2: **POD_OF_RHS**(B, \cdot)
- 3: **while** $E_{\max} > tol$ AND no. columns of $Q_G < q_{\max}$ **do**
- 4: $[Q_G, q, \cdot] = \text{SPARAMSAPOR}(q, f, \Delta, Q_G, tol, N_{\max}^{\text{moments}}, \cdot)$
- 5: **UPDATEMATRICES**(Q_G, \cdot)
- 6: $[E_{\max}, f_{E_{\max}}] = \text{ESTERROR}(f_{\min}, f_{\max}, \cdot)$;
- 7: Set the new expansion point $f = f_{E_{\max}}, j = j + 1$
- 8: **if** $j == 1$ **then** $\Delta = \text{FINDSUBRANGESIZE}(tol, \cdot)$
- 9: **end while**
- 10: **return** C_r, G_r, Γ_r and B_r

The second step (**POD_OF_RHS**(B, \cdot)) is performed to analyse the structures, which are excited through the ports that exhibit non-affine nature (micro-strip, coplanar wave-guides etc.). The details of this technique is provided in [2].

Algorithm 2: SPARAMSAPOR: Single-point block SAPOR

Require: $q, f, \Delta, Q_G, tol, N_{\max}^{\text{moments}}, B^T B, B^T C Q_G, B^T G Q_G, B^T \Gamma Q_G$

- 1: For $s_0 = j2\pi f$ compute Q_1, P_1 .
- 2: $q \leftarrow q + 1, Q_{G,q} = Q_1$
- 3: **MODIFIEDGRAMSCHMIDT**($Q_{G,q}$)
- 4: **UPDATEMATRICES**($Q_{G,q}, \cdot$)
- 5: $[E_{\max}^{\text{loc}}, \cdot] = \text{ESTERROR}(f - \Delta/2, f + \Delta/2, \cdot)$
- 6: Set $i = 1$
- 7: **while** $E_{\max}^{\text{loc}} > tol$ AND $i < N_{\max}^{\text{moments}}$ **do**
- 8: For $s_0 = j2\pi f$ compute Q_{i+1}, P_{i+1} .
- 9: $q \leftarrow q + 1, Q_{G,q} = Q_{i+1}$
- 10: **MODIFIEDGRAMSCHMIDT**($Q_{G,q}$)
- 11: **UPDATEMATRICES**($Q_{G,q}, \cdot$)
- 12: $[E_{\max}^{\text{loc}}, \cdot] = \text{ESTERROR}(f - \Delta/2, f + \Delta/2, \cdot)$
- 13: Set $i \leftarrow i + 1$
- 14: **end while**
- 15: **return** $Q_G, q, B^T C Q_G, B^T G Q_G$ and $B^T \Gamma Q_G$

2 Simulations

All tests have been performed on Intel Core i5-6500 CPU @3.20GHz, 64GM RAM Workstation using C++ implementation. For each of the structures we provide the parameters of simulation (number of variables, number of frequency points) error and estimator plots, as well as the reduction parameters: computational time (divided into parts of the algorithm), approximated speed-up, number of vectors in \mathbf{Q} , number of expansion frequency points (in which the subsequent block moments are constructed). The steps of the algorithms 1 and 2 associated with the corresponding operations are listed in the table (in bold).

The speed-up (SU) of computations is computed as follows:

$$SU = \frac{t_{\text{solve}FEM}}{t_{MOR} + t_{\text{solve}MOR}} \quad (1)$$

where $t_{\text{solve}FEM}$ is the computational time of the direct sweep (based on the original FEM model), t_{MOR} is the total model-order-reduction time, and finally, $t_{\text{solve}MOR}$ is the time of the reduced order model evaluation in the whole frequency bandwidth.

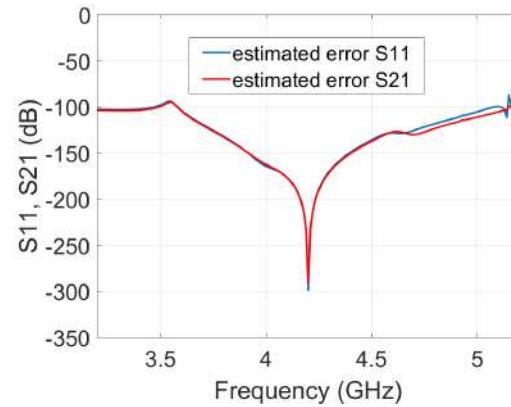
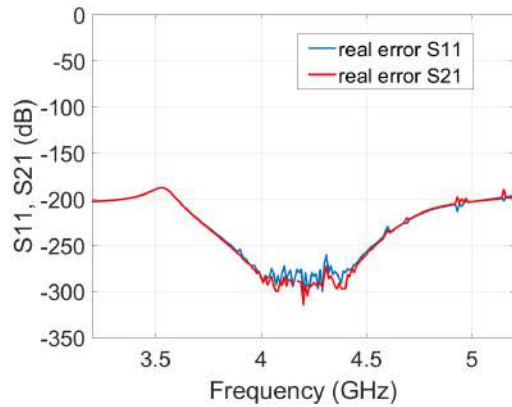
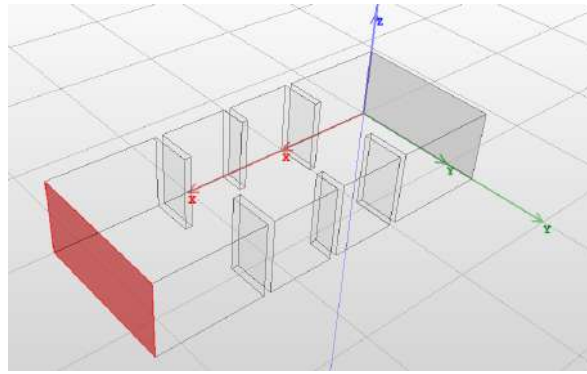
The positions in the tables denotes:

- SF/NF/Solv - computational time of the symbolical factorization, numerical factorization and the solution based on the factorants.
- Lossy - is the problem lossy? The dispersive surface boundary conditions are modeled using the formulation published in [3].
- Solution - cumulated solution times in RGM-MOR (with numerical and symbolical factorizations)

References

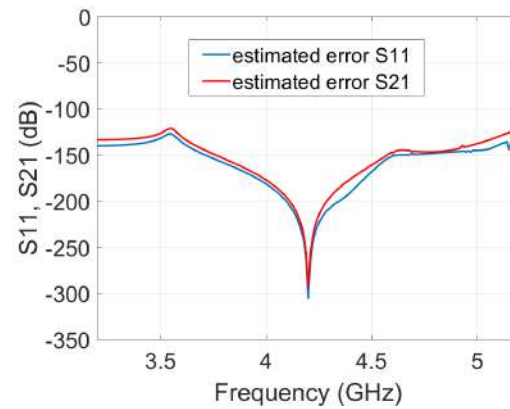
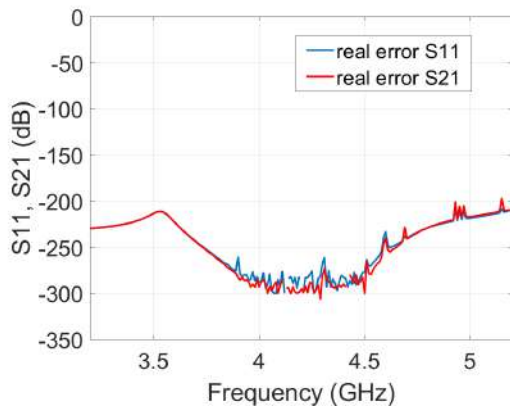
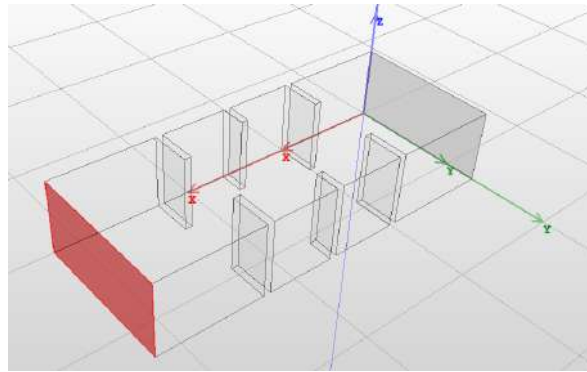
- [1] Fotyga, G., Czarniewska, M., Lamecki, A., & Mrozowski, M. (2018). *Reliable Greedy Multipoint Model-Order Reduction Techniques for Finite Element Analysis*. IEEE Antennas and Wireless Propagation Letters.
- [2] Fotyga, G., *EDISON Raport 1. Multipoint Reduction Approach for Non-Affine Right Hand Side Problems*, 2017.
- [3] Rewieński, M., Lamecki, A., & Mrozowski, M. (2015). *Model order reduction for problems with dispersive surface boundary conditions*. IEEE Microwave and Wireless Components Letters, 25(9), 561-563.

2.1 Lossy iris filter (tolerance 1e-4)



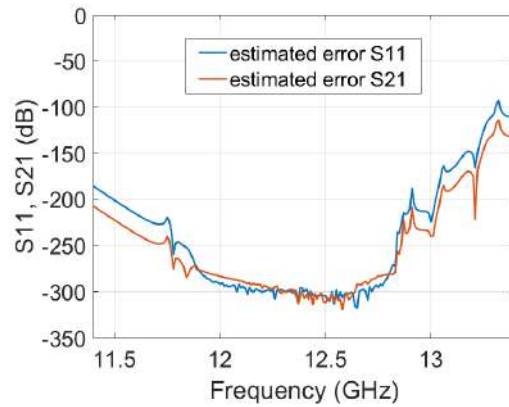
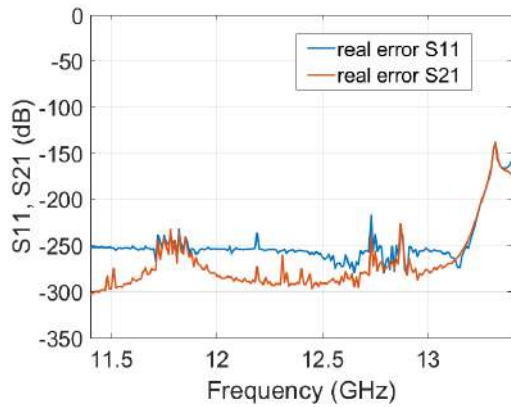
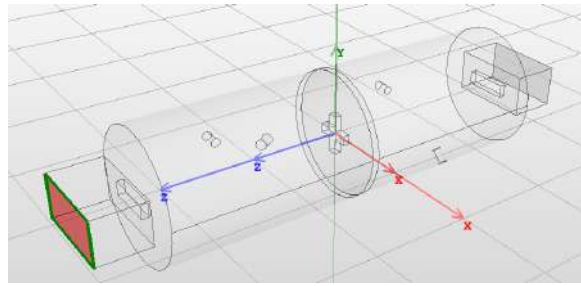
Approx. speed - up	1.724
Reduction time	1.196
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.002
SF/NF/Solv	0.052 / 0.007 / 0.003
Solution (A2, steps: 1,8)	0.148
Orthogonalization (inside SAPOR) (A2, steps 3,10)	0.061
Local estimator (A2, step 12)	0.121
Global estimator (A1, step 6)	0.04
Update matrix (A1 step 5; A2, steps 4, 11)	0.029
Final frequency sweep	0.781
Number of vectors in the basis	44
Number of expansion freq. points	1
Number of variables	5846
Number of frequency points	201
Lossy	1
Absorbing Boundary conditions	0

2.2 Lossy iris filter (tolerance 1e-6)



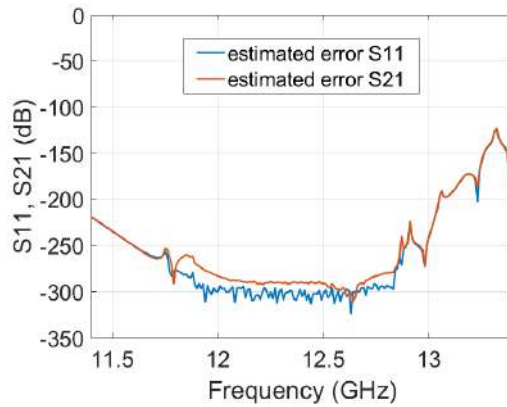
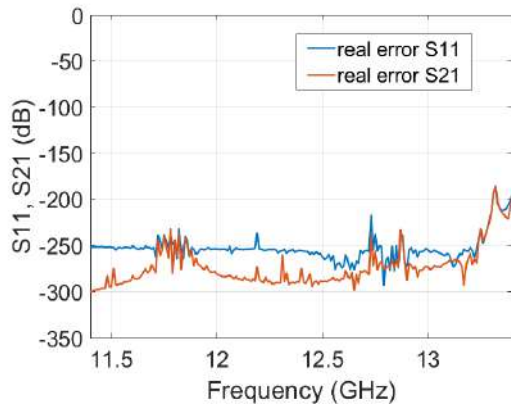
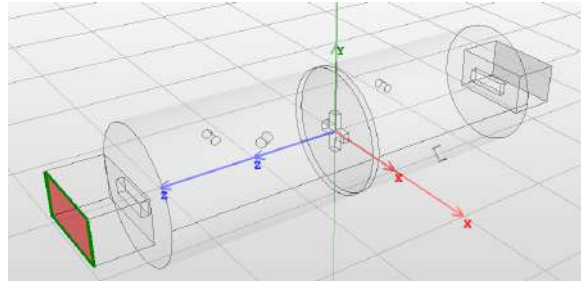
Approx. speed - up	0.952
Reduction time	2.165
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.001
SF/NF/Solv	0.05 / 0.008 / 0.002
Solution (A2, steps: 1,8)	0.173
Orthogonalization (inside SAPOR) (A2, steps 3,10)	0.126
Local estimator (A2, step 12)	0.608
Global estimator (A1, step 6)	0.093
Update matrix (A1 step 5; A2, steps 4, 11)	0.055
Final frequency sweep	1.092
Number of vectors in the basis	62
Number of expansion freq. points	1
Number of variables	5846
Number of frequency points	201
Lossy	1
Absorbing Boundary conditions	0

2.3 Dual mode filter (tolerance $1e-4$)



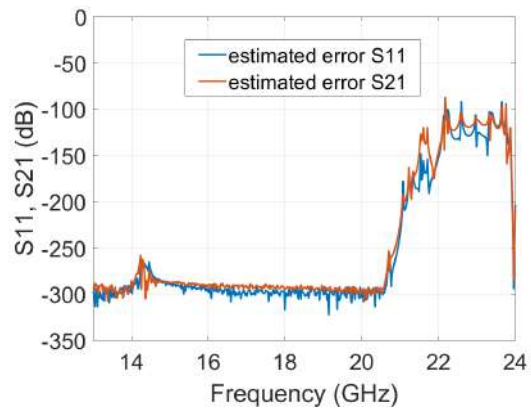
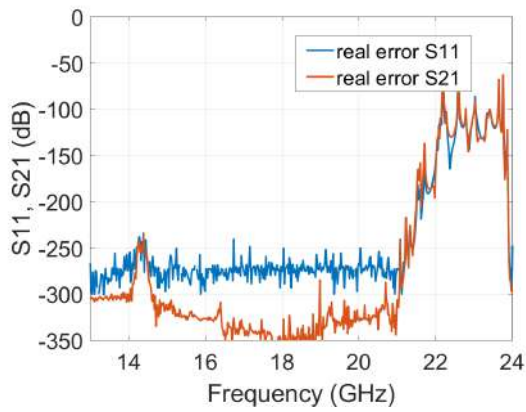
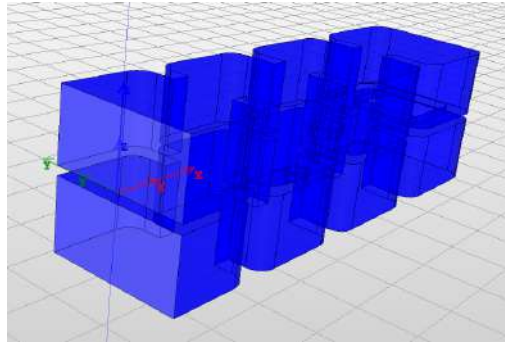
Approx. speed - up	11.742
Reduction time	4.352
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.015
SF/NF/Solv	0.851 / 0.209 / 0.041
Solution (A2, steps: 1,8)	2.399
Orthogonalization (inside SAPOR) (A2, steps 3,10)	0.948
Local estimator (A2, step 12)	0.214
Global estimator (A1, step 6)	0.058
Update matrix (A1 step 5; A2, steps 4, 11)	0.611
Final frequency sweep	0.02
Number of vectors in the basis	48
Number of expansion freq. points	1
Number of variables	82662
Number of frequency points	201
Lossy	0
Absorbing Boundary conditions	0

2.4 Dual mode filter (tolerance 1e-6)



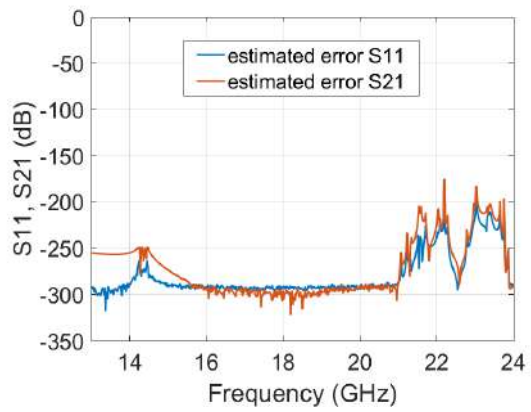
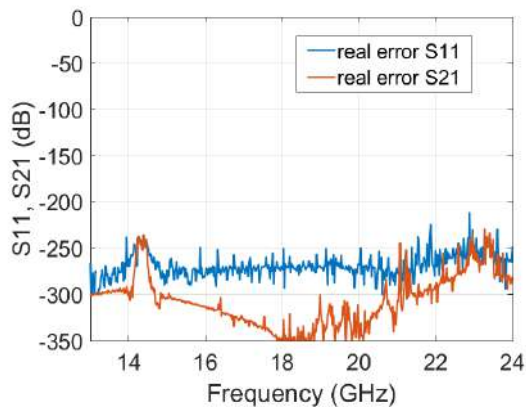
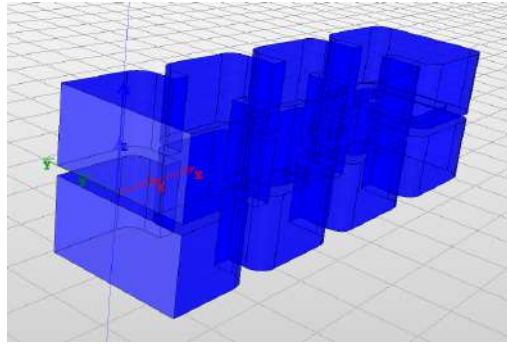
Approx. speed - up	14.414
Reduction time	4.397
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.017
SF/NF/Solv	0.866 / 0.266 / 0.045
Solution (A2, steps: 1,8)	2.556
Orthogonalization (inside SAPOR) (A2, steps 3,10)	1.003
Local estimator (A2, step 12)	0.121
Global estimator (A1, step 6)	0.064
Update matrix (A1 step 5; A2, steps 4, 11)	0.526
Final frequency sweep	0.022
Number of vectors in the basis	50
Number of expansion freq. points	1
Number of variables	82662
Number of frequency points	201
Lossy	0
Absorbing Boundary conditions	0

2.5 Folded filter (tolerance 1e-4)



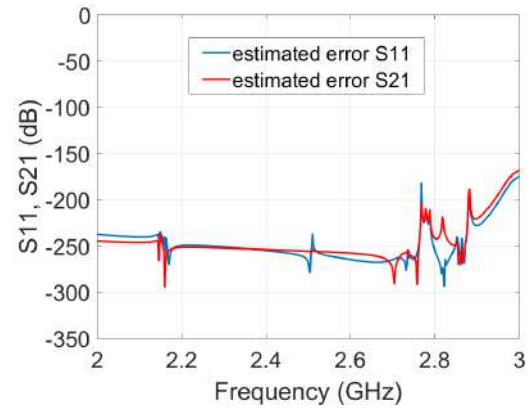
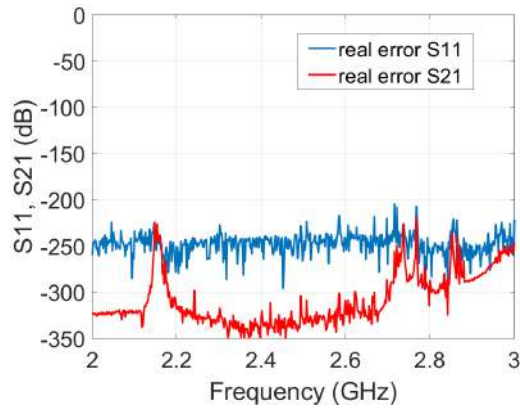
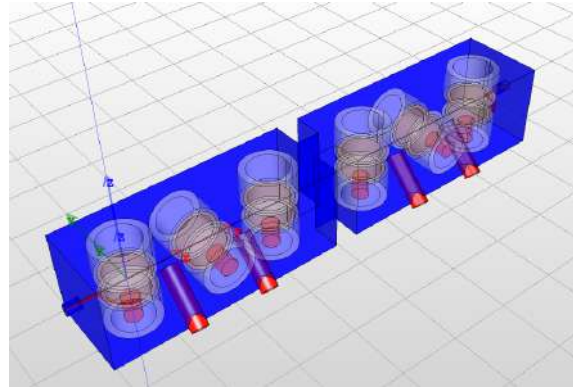
Approx. speed - up	44.458
Reduction time	94.806
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.151
SF/NF/Solv	9.977 / 7.513 / 0.586
Solution (A2, steps: 1,8)	54.09
Orthogonalization (inside SAPOR) (A2, steps 3,10)	28.35
Local estimator (A2, step 12)	0.426
Global estimator (A1, step 6)	0.57
Update matrix (A1 step 5; A2, steps 4, 11)	8.616
Final frequency sweep	0.157
Number of vectors in the basis	74
Number of expansion freq. points	2
Number of variables	864958
Number of frequency points	501
Lossy	0
Absorbing Boundary conditions	0

2.6 Folded filter (tolerance 1e-6)



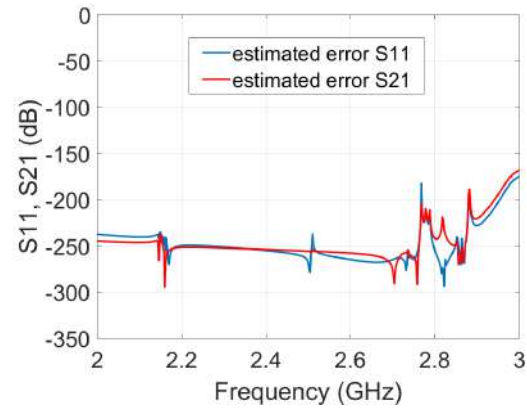
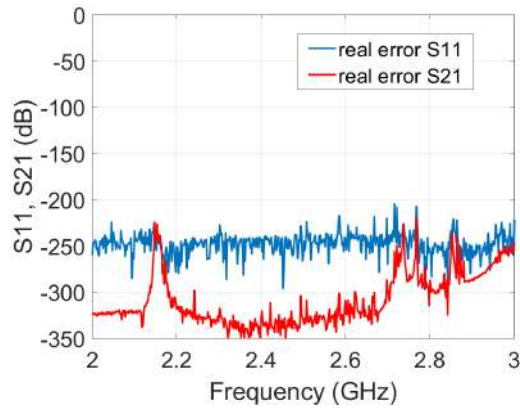
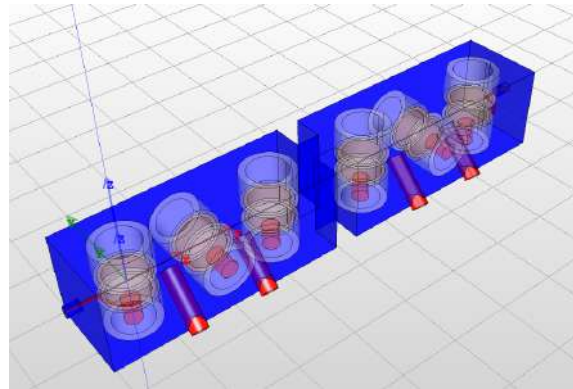
Approx. speed - up	33.288
Reduction time	125.554
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.148
SF/NF/Solv	10.14 / 7.467 / 0.56
Solution (A2, steps: 1,8)	67.87
Orthogonalization (inside SAPOR) (A2, steps 3,10)	42.03
Local estimator (A2, step 12)	0.47
Global estimator (A1, step 6)	1.114
Update matrix (A1 step 5; A2, steps 4, 11)	11.45
Final frequency sweep	0.179
Number of vectors in the basis	92
Number of expansion freq. points	3
Number of variables	864958
Number of frequency points	501
Lossy	0
Absorbing Boundary conditions	0

2.7 Simone filter (tolerance 1e-4)



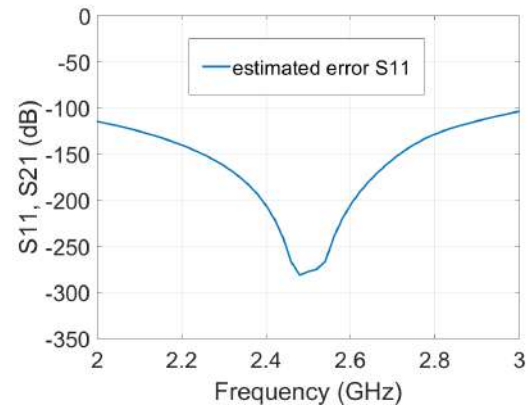
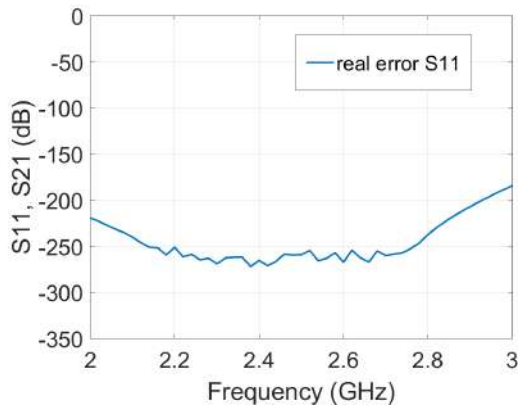
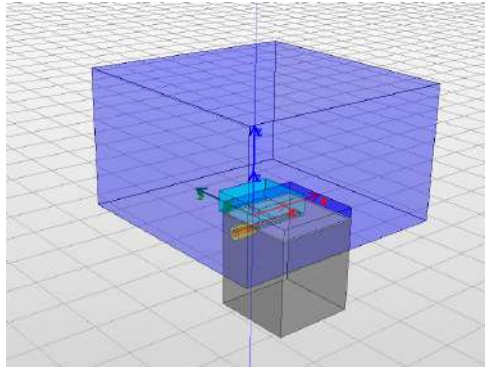
Approx. speed - up	33.508
Reduction time	35.296
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.037
SF/NF/Solv	3.855 / 2.21 / 0.224
Solution (A2, steps: 1,8)	19.14
Orthogonalization (inside SAPOR) (A2, steps 3,10)	10.69
Local estimator (A2, step 12)	0.541
Global estimator (A1, step 6)	0.515
Update matrix (A1 step 5; A2, steps 4, 11)	3.775
Final frequency sweep	0.107
Number of vectors in the basis	74
Number of expansion freq. points	2
Number of variables	348028
Number of frequency points	501
Lossy	0
Absorbing Boundary conditions	0

2.8 Simone filter (tolerance 1e-6)



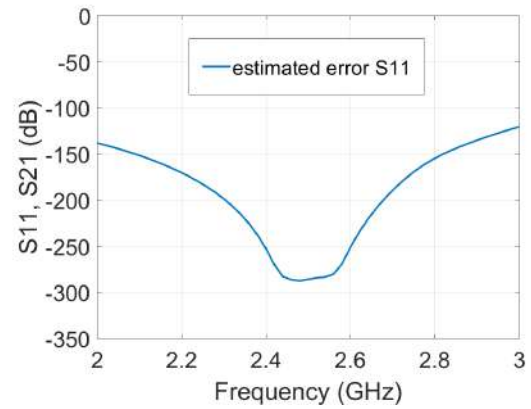
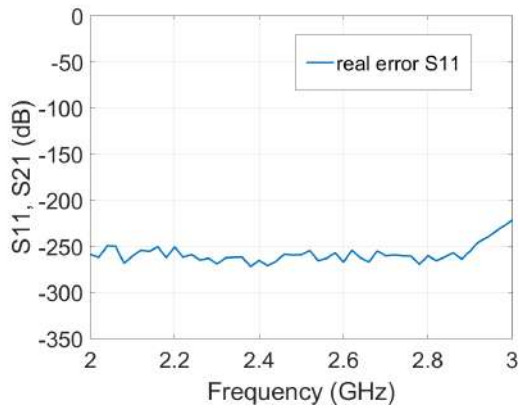
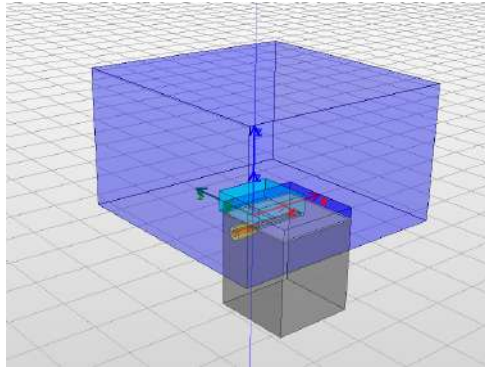
Approx. speed - up	33.803
Reduction time	35.597
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.035
SF/NF/Solv	3.888 / 2.035 / 0.21
Solution (A2, steps: 1,8)	18.7
Orthogonalization (inside SAPOR) (A2, steps 3,10)	10.44
Local estimator (A2, step 12)	0.508
Global estimator (A1, step 6)	1.35
Update matrix (A1 step 5; A2, steps 4, 11)	3.373
Final frequency sweep	0.427
Number of vectors in the basis	74
Number of expansion freq. points	2
Number of variables	348028
Number of frequency points	501
Lossy	0
Absorbing Boundary conditions	0

2.9 DRA antenna (tolerance 1e-4)



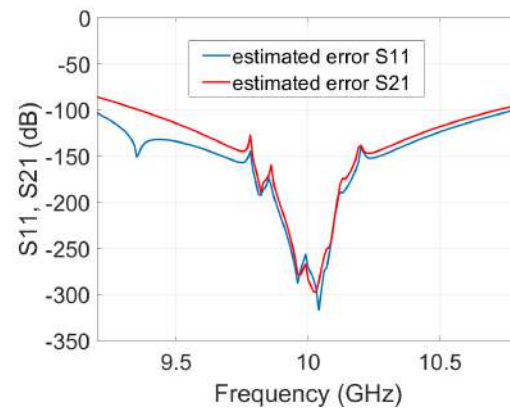
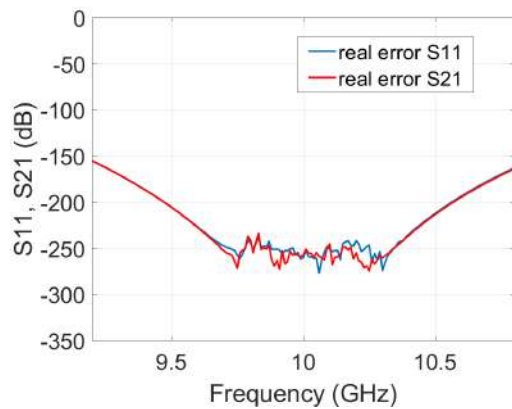
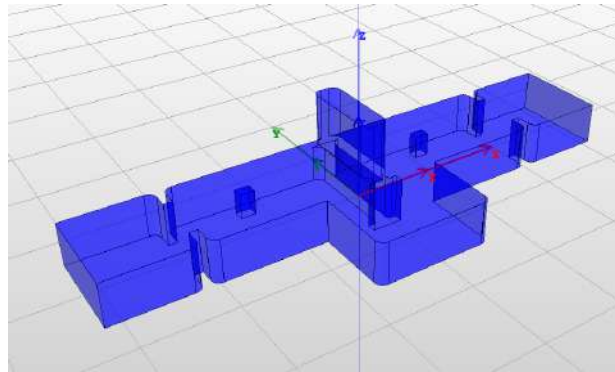
Approx. speed - up	16.729
Reduction time	8.500
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.006
SF/NF/Solv	2.101 / 2.615 / 0.132
Solution (A2, steps: 1,8)	6.647
Orthogonalization (inside SAPOR) (A2, steps 3,10)	0.175
Local estimator (A2, step 12)	0.002
Global estimator (A1, step 6)	0.001
Update matrix (A1 step 5; A2, steps 4, 11)	0.416
Final frequency sweep	1.108
Number of vectors in the basis	8
Number of expansion freq. points	1
Number of variables	169478
Number of frequency points	51
Lossy	0
Absorbing Boundary conditions	1

2.10 DRA antenna (tolerance 1e-6)



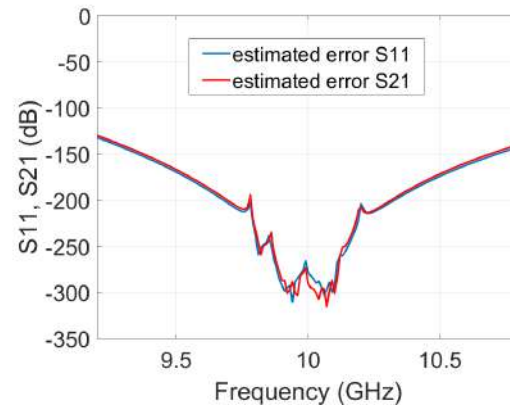
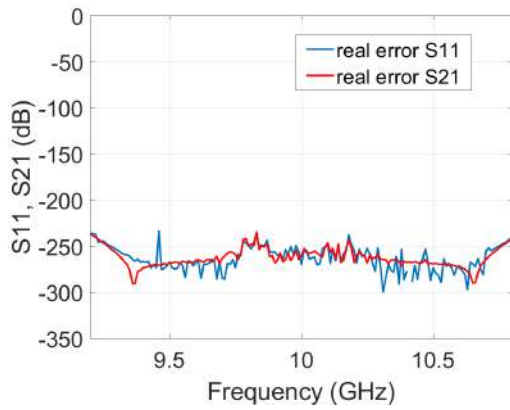
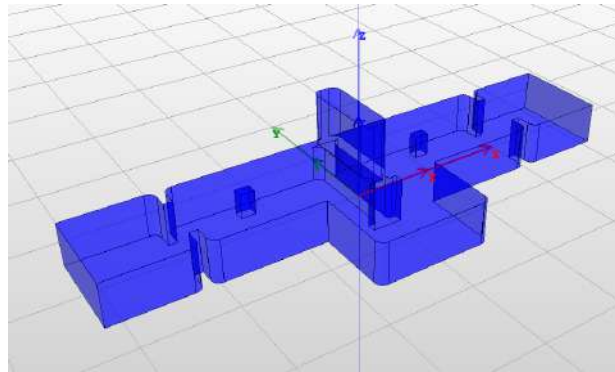
Approx. speed - up	15.159
Reduction time	9.242
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.007
SF/NF/Solv	2.141 / 2.576 / 0.129
Solution (A2, steps: 1,8)	7.098
Orthogonalization (inside SAPOR) (A2, steps 3,10)	0.25
Local estimator (A2, step 12)	0.002
Global estimator (A1, step 6)	0.001
Update matrix (A1 step 5; A2, steps 4, 11)	0.441
Final frequency sweep	1.298
Number of vectors in the basis	10
Number of expansion freq. points	1
Number of variables	169478
Number of frequency points	51
Lossy	0
Absorbing Boundary conditions	1

2.11 Szydowski filter (tolerance 1e-4)



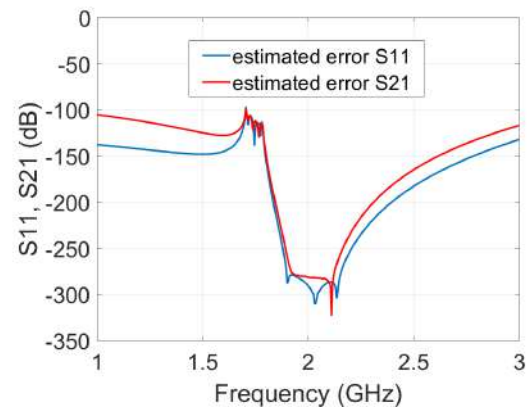
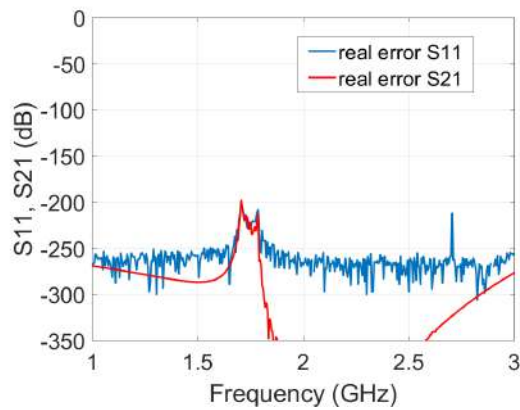
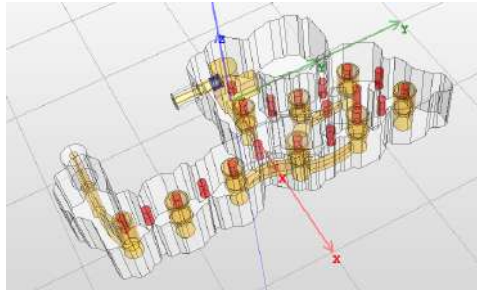
Approx. speed - up	37.001
Reduction time	25.768
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.14
SF/NF/Solv	9.005 / 5.38 / 0.486
Solution (A2, steps: 1,8)	20.64
Orthogonalization (inside SAPOR) (A2, steps 3,10)	2.166
Local estimator (A2, step 12)	0.013
Global estimator (A1, step 6)	0.008
Update matrix (A1 step 5; A2, steps 4, 11)	2.188
Final frequency sweep	0.032
Number of vectors in the basis	18
Number of expansion freq. points	1
Number of variables	815370
Number of frequency points	161
Lossy	0
Absorbing Boundary conditions	0

2.12 Szydlowski filter (tolerance 1e-6)



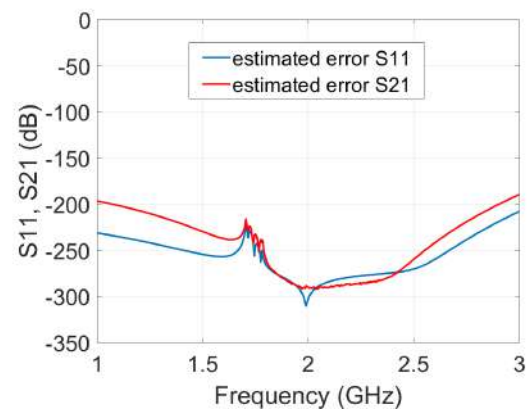
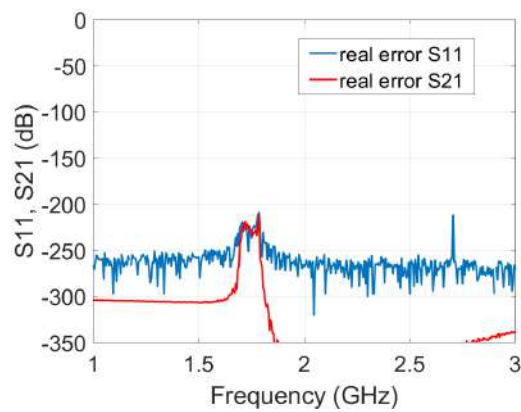
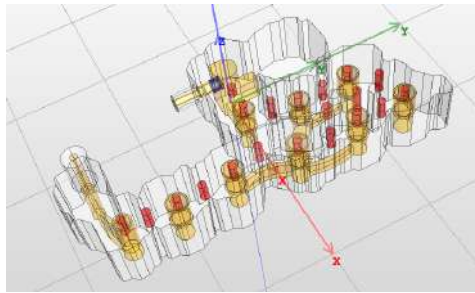
Approx. speed - up	33.522
Reduction time	28.263
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.138
SF/NF/Solv	9.125 / 5.36 / 0.467
Solution (A2, steps: 1,8)	21.93
Orthogonalization (inside SAPOR) (A2, steps 3,10)	2.974
Local estimator (A2, step 12)	0.016
Global estimator (A1, step 6)	0.01
Update matrix (A1 step 5; A2, steps 4, 11)	2.504
Final frequency sweep	0.034
Number of vectors in the basis	22
Number of expansion freq. points	1
Number of variables	815370
Number of frequency points	161
Lossy	0
Absorbing Boundary conditions	0

2.13 GSM filter (tolerance 1e-4)



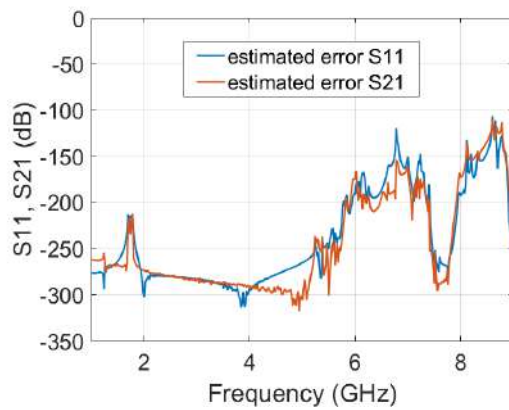
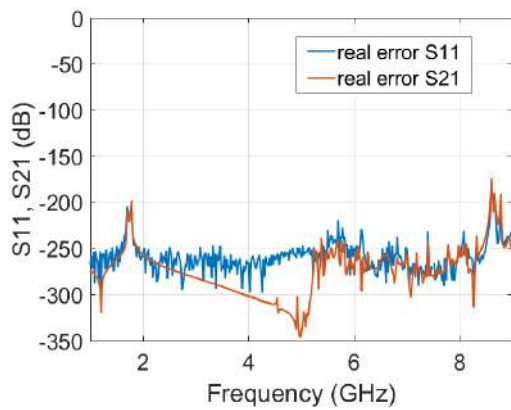
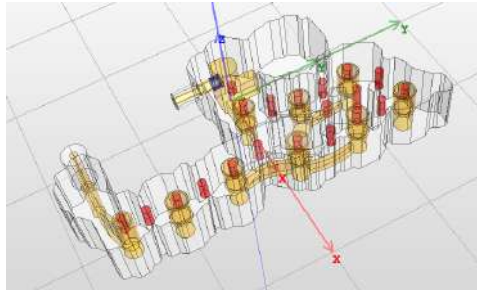
Approx. speed - up	25.588
Reduction time	18.017
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.067
SF/NF/Solv	7.089 / 0.842 / 0.29
Solution (A2, steps: 1,8)	12.78
Orthogonalization (inside SAPOR) (A2, steps 3,10)	2.546
Local estimator (A2, step 12)	0.037
Global estimator (A1, step 6)	0.023
Update matrix (A1 step 5; A2, steps 4, 11)	2.11
Final frequency sweep	0.035
Number of vectors in the basis	22
Number of expansion freq. points	1
Number of variables	709372
Number of frequency points	401
Lossy	0
Absorbing Boundary conditions	0

2.14 GSM filter (tolerance 1e-6)



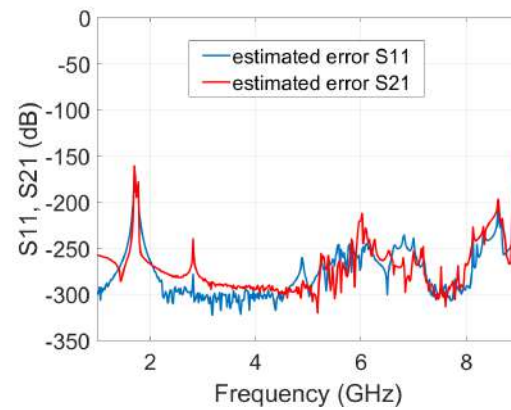
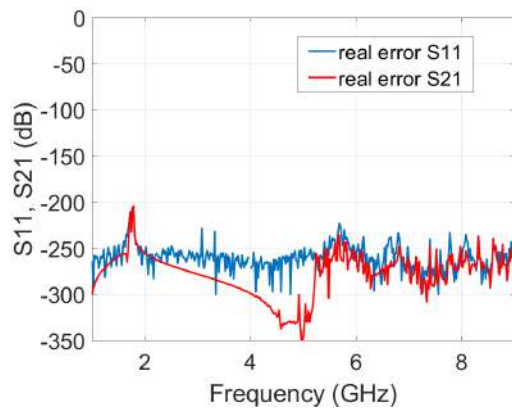
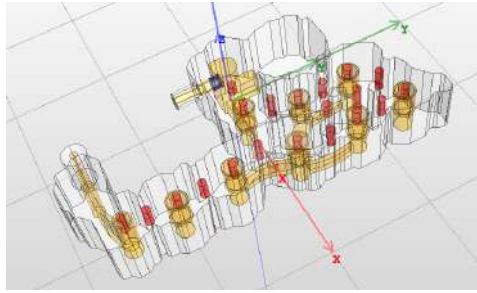
Approx. speed - up	21.919
Reduction time	20.637
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.068
SF/NF/Solv	6.838 / 0.829 / 0.282
Solution (A2, steps: 1,8)	13.53
Orthogonalization (inside SAPOR) (A2, steps 3,10)	3.954
Local estimator (A2, step 12)	0.043
Global estimator (A1, step 6)	0.03
Update matrix (A1 step 5; A2, steps 4, 11)	2.49
Final frequency sweep	0.039
Number of vectors in the basis	28
Number of expansion freq. points	1
Number of variables	709372
Number of frequency points	401
Lossy	0
Absorbing Boundary conditions	0

2.15 GSM filter - wideband (tolerance 1e-4)



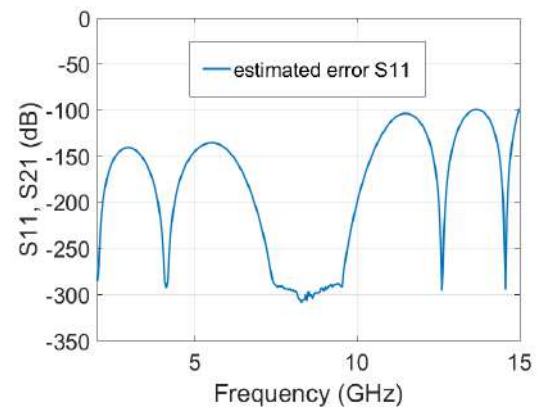
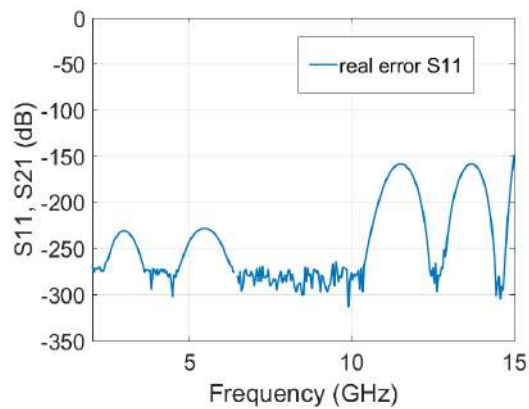
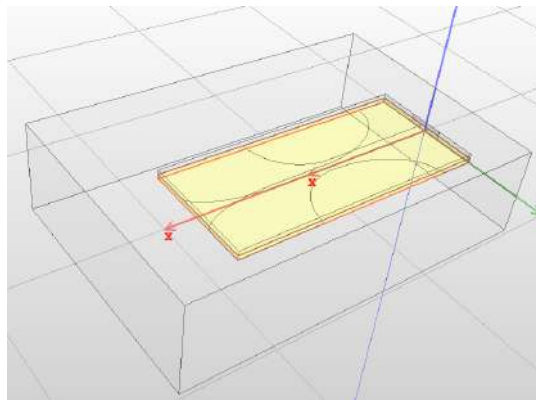
Approx. speed - up	5.435
Reduction time	113.729
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.086
SF/NF/Solv	8.578 / 1.007 / 0.334
Solution (A2, steps: 1,8)	41.54
Orthogonalization (inside SAPOR) (A2, steps 3,10)	55.26
Local estimator (A2, step 12)	0.44
Global estimator (A1, step 6)	1.342
Update matrix (A1 step 5; A2, steps 4, 11)	12.68
Final frequency sweep	0.174
Number of vectors in the basis	108
Number of expansion freq. points	4
Number of variables	855078
Number of frequency points	401
Lossy	0
Absorbing Boundary conditions	0

2.16 GSM filter - wideband (tolerance 1e-6)



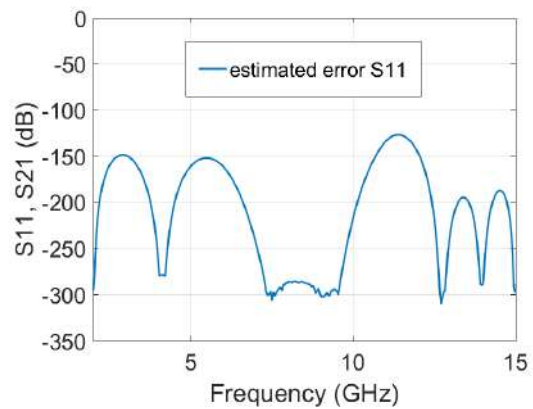
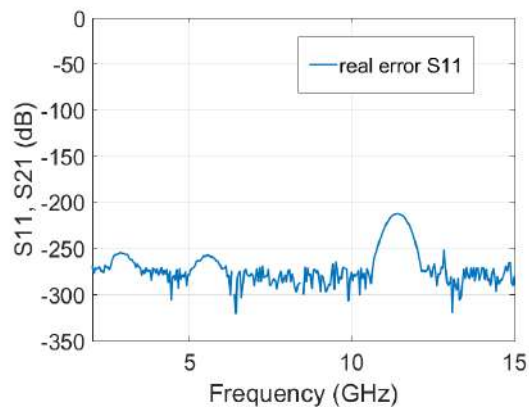
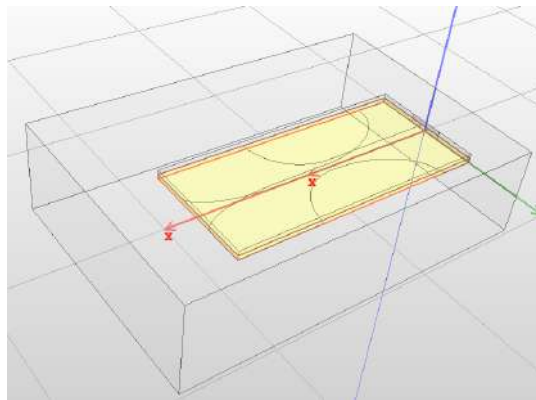
Approx. speed - up	4.961
Reduction time	122.814
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.082
SF/NF/Solv	8.529 / 1.013 / 0.326
Solution (A2, steps: 1,8)	42.71
Orthogonalization (inside SAPOR) (A2, steps 3,10)	62.28
Local estimator (A2, step 12)	0.419
Global estimator (A1, step 6)	2.068
Update matrix (A1 step 5; A2, steps 4, 11)	12.86
Final frequency sweep	0.192
Number of vectors in the basis	114
Number of expansion freq. points	4
Number of variables	855078
Number of frequency points	401
Lossy	0
Absorbing Boundary conditions	0

2.17 Vivaldi Antenna (tolerance 1e-4)



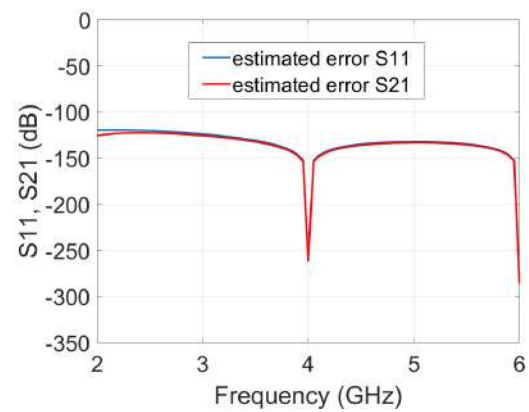
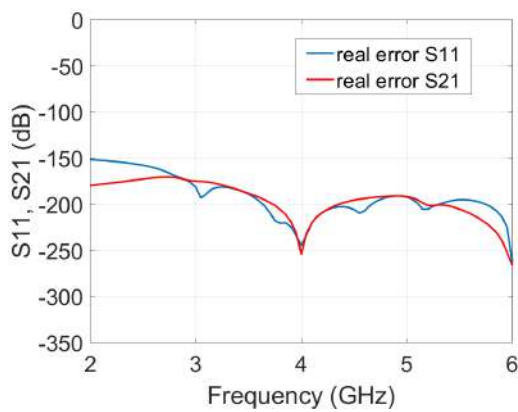
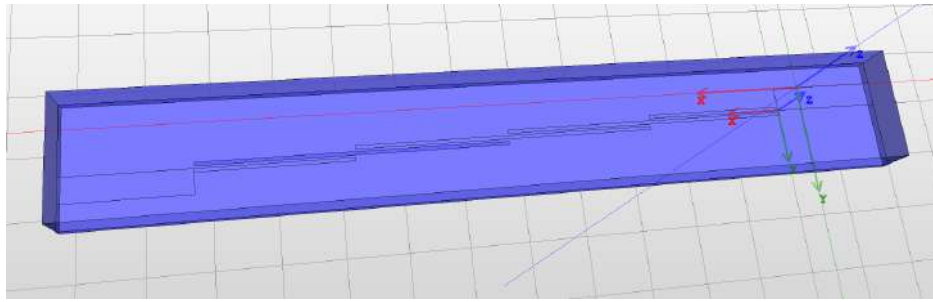
Approx. speed - up	35.526
Reduction time	134.803
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.015
SF/NF/Solv	4.897 / 15.23 / 0.422
Solution (A2, steps: 1,8)	118.1
Orthogonalization (inside SAPOR) (A2, steps 3,10)	9.973
Local estimator (A2, step 12)	0.068
Global estimator (A1, step 6)	0.193
Update matrix (A1 step 5; A2, steps 4, 11)	4.286
Final frequency sweep	1.116
Number of vectors in the basis	55
Number of expansion freq. points	5
Number of variables	370474
Number of frequency points	301
Lossy	0
Absorbing Boundary conditions	1

2.18 Vivaldi Antenna (tolerance 1e-6)



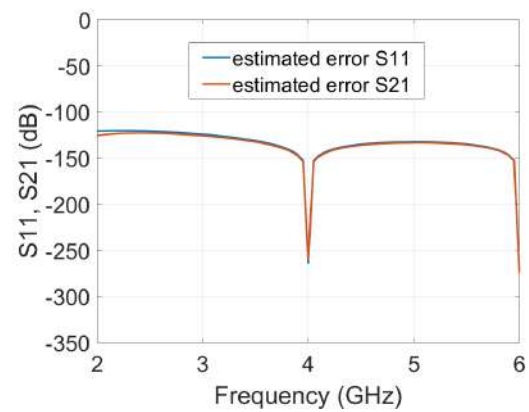
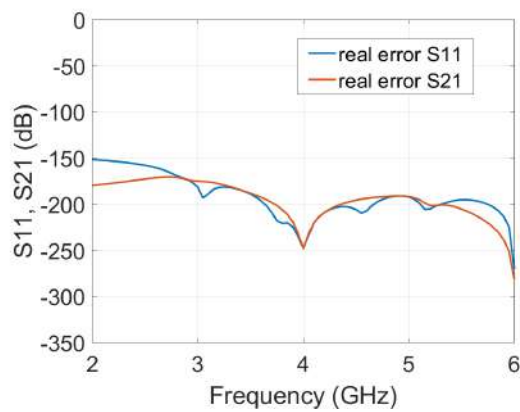
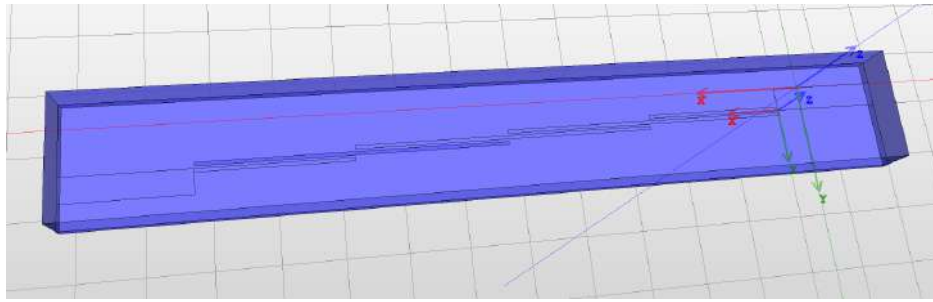
Approx. speed - up	28.332
Reduction time	168.904
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.014
SF/NF/Solv	5.124 / 17.18 / 0.486
Solution (A2, steps: 1,8)	147
Orthogonalization (inside SAPOR) (A2, steps 3,10)	13.32
Local estimator (A2, step 12)	0.077
Global estimator (A1, step 6)	0.261
Update matrix (A1 step 5; A2, steps 4, 11)	5.715
Final frequency sweep	1.351
Number of vectors in the basis	65
Number of expansion freq. points	6
Number of variables	370474
Number of frequency points	301
Lossy	0
Absorbing Boundary conditions	1

2.19 Coupled Transmission Line Filter (tolerance $1e-4$)



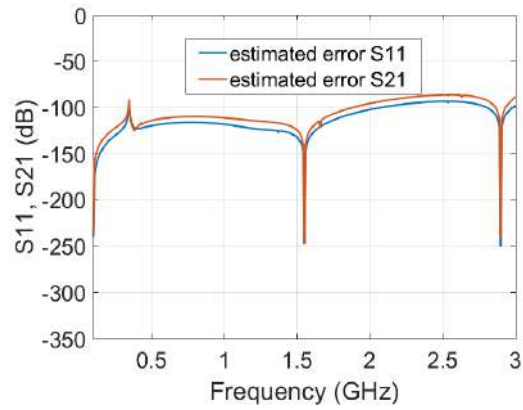
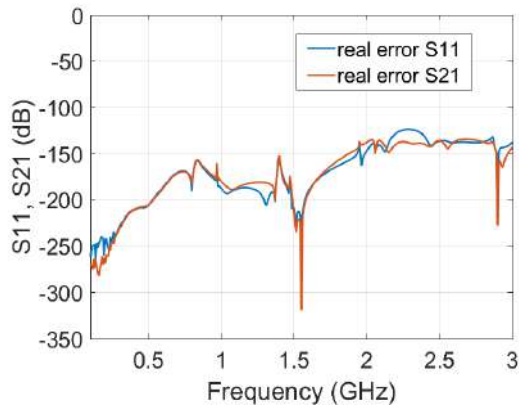
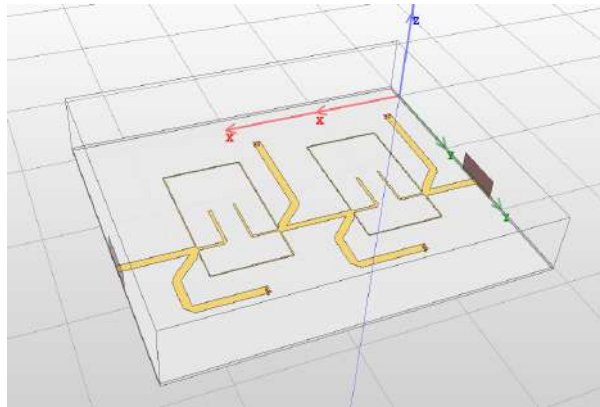
Approx. speed - up	41.990
Reduction time	244.392
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	19.34
SF/NF/Solv	13.06 / 32.2 / 1.539
Solution (A2, steps: 1,8)	150.2
Orthogonalization (inside SAPOR) (A2, steps 3,10)	40.99
Local estimator (A2, step 12)	1.294
Global estimator (A1, step 6)	0.748
Update matrix (A1 step 5; A2, steps 4, 11)	19.7
Final frequency sweep	10.07
Number of vectors in the basis	74
Number of expansion freq. points	2
Number of variables	926510
Number of frequency points	300
Lossy	0
Absorbing Boundary conditions	1

2.20 Coupled Transmission Line Filter (tolerance 1e-6)



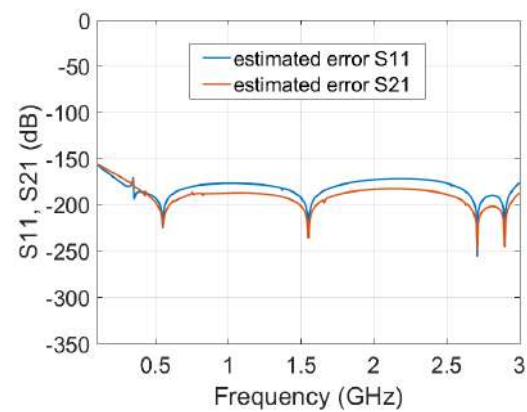
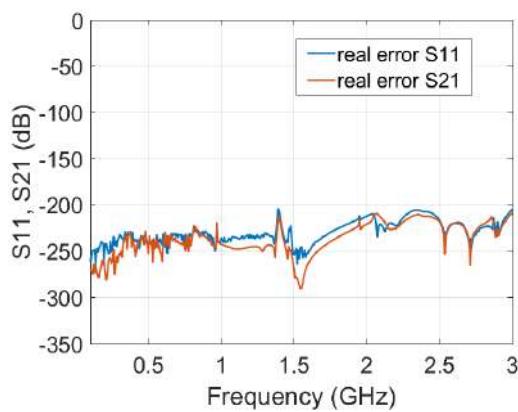
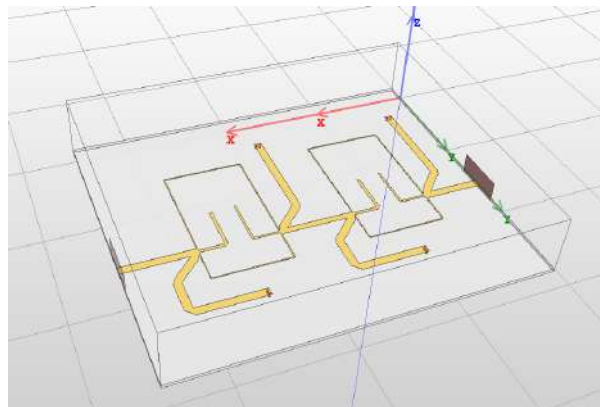
Approx. speed - up	46.106
Reduction time	242.049
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	20.37
SF/NF/Solv	14.12 / 32.24 / 1.424
Solution (A2, steps: 1,8)	154.4
Orthogonalization (inside SAPOR) (A2, steps 3,10)	41.06
Local estimator (A2, step 12)	0.355
Global estimator (A1, step 6)	0.377
Update matrix (A1 step 5; A2, steps 4, 11)	12.41
Final frequency sweep	9.77
Number of vectors in the basis	74
Number of expansion freq. points	2
Number of variables	926510
Number of frequency points	300
Lossy	0
Absorbing Boundary conditions	1

2.21 Filter Roberto (tolerance 1e-4)



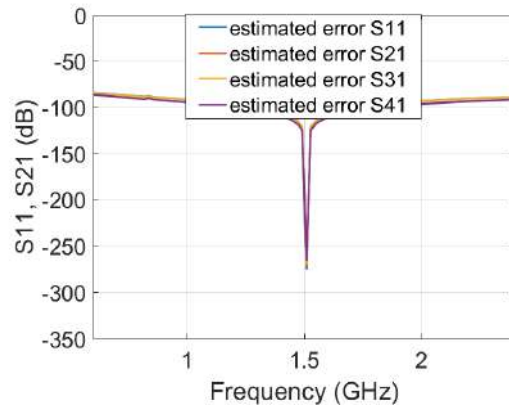
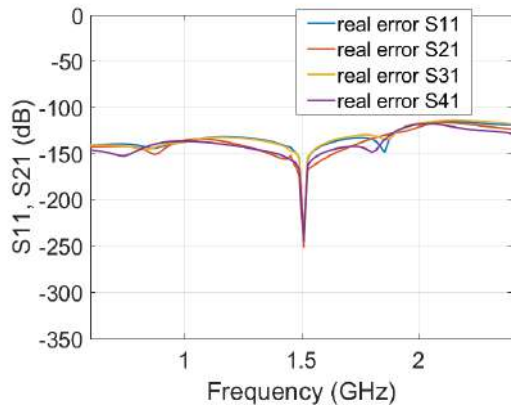
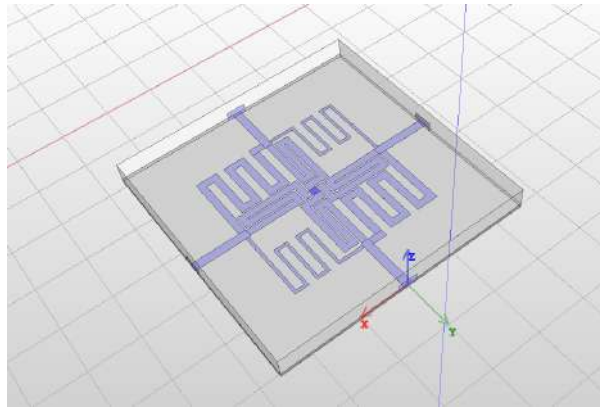
Approx. speed - up	33.630
Reduction time	147.489
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	13.76
SF/NF/Solv	6.94 / 11.29 / 0.628
Solution (A2, steps: 1,8)	84.39
Orthogonalization (inside SAPOR) (A2, steps 3,10)	30.3
Local estimator (A2, step 12)	0.559
Global estimator (A1, step 6)	0.997
Update matrix (A1 step 5; A2, steps 4, 11)	8.217
Final frequency sweep	7.918
Number of vectors in the basis	86
Number of expansion freq. points	3
Number of variables	552226
Number of frequency points	401
Lossy	0
Absorbing Boundary conditions	1

2.22 Filter Roberto (tolerance 1e-6)



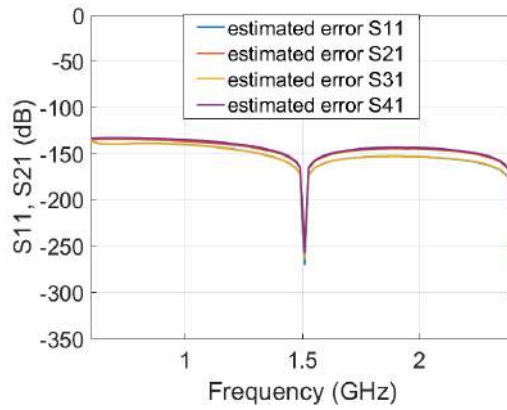
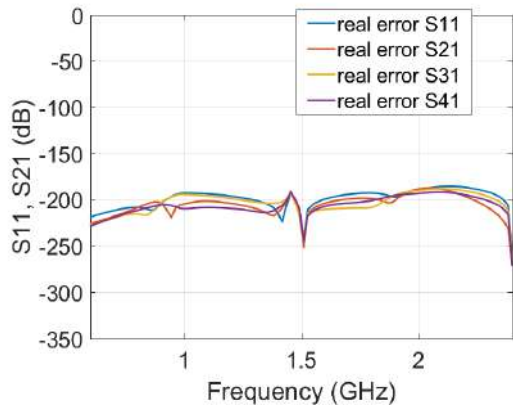
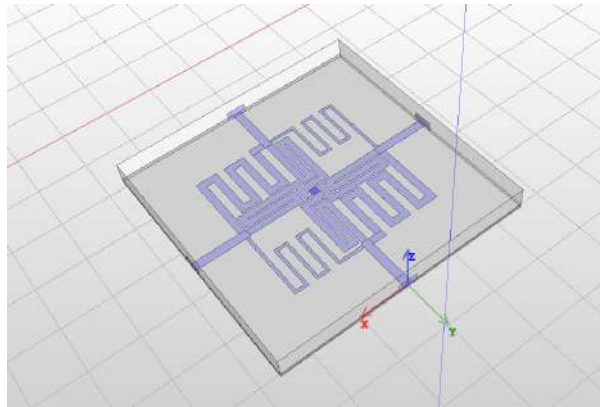
Approx. speed - up	19.836
Reduction time	248.218
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	12.74
SF/NF/Solv	6.902 / 11.28 / 0.602
Solution (A2, steps: 1,8)	124.2
Orthogonalization (inside SAPOR) (A2, steps 3,10)	77.92
Local estimator (A2, step 12)	1.111
Global estimator (A1, step 6)	2.42
Update matrix (A1 step 5; A2, steps 4, 11)	16.43
Final frequency sweep	11.08
Number of vectors in the basis	148
Number of expansion freq. points	4
Number of variables	552226
Number of frequency points	401
Lossy	0
Absorbing Boundary conditions	1

2.23 Branch line Coupler (tolerance 1e-4)



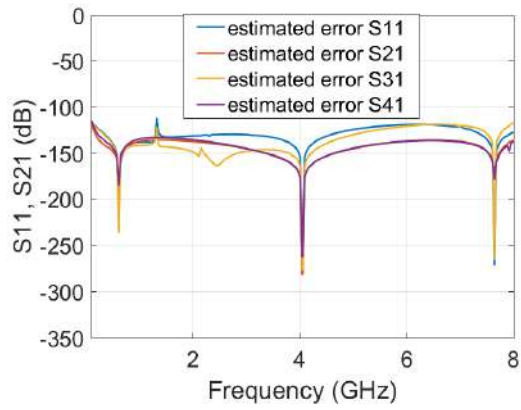
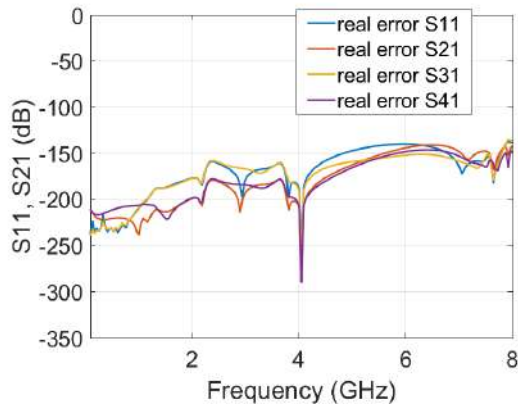
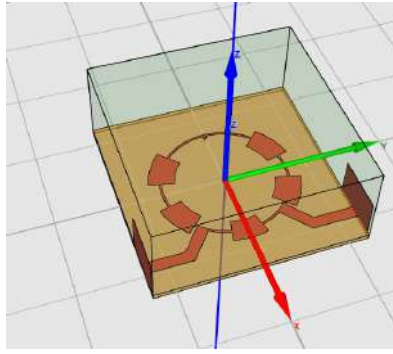
Approx. speed - up	21.847
Reduction time	44.253
Tolerance	0.0001
Max. number of moments/freq.point	16
Initial (POD of RHS) (A1, step 2)	2.741
SF/NF/Solv	5.379 / 8.492 / 1.04
Solution (A2, steps: 1,8)	27.81
Orthogonalization (inside SAPOR) (A2, steps 3,10)	8.013
Local estimator (A2, step 12)	0.09
Global estimator (A1, step 6)	0.039
Update matrix (A1 step 5; A2, steps 4, 11)	2.829
Final frequency sweep	2.147
Number of vectors in the basis	44
Number of expansion freq. points	1
Number of variables	385588
Number of frequency points	100
Lossy	0
Absorbing Boundary conditions	1

2.24 Branch line Coupler (tolerance 1e-6)



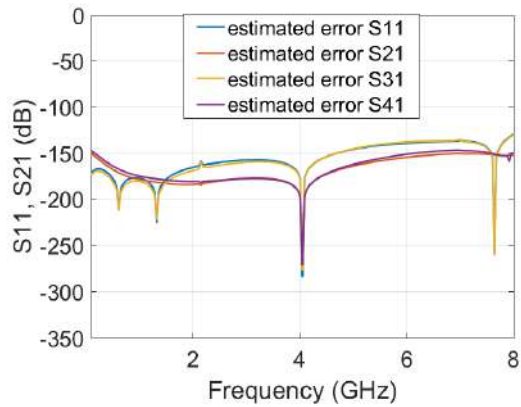
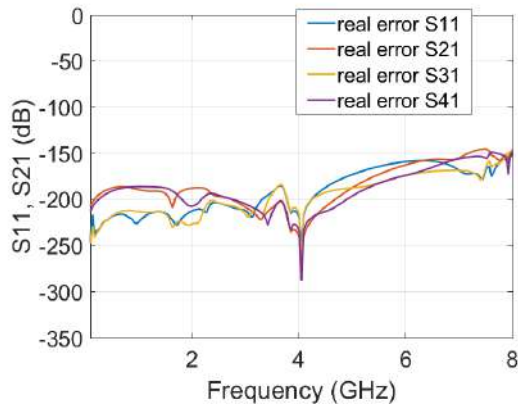
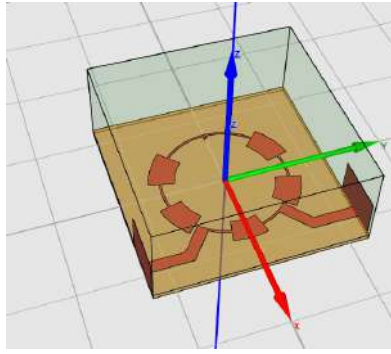
Approx. speed - up	10.970
Reduction time	72.906
Tolerance	1e-006
Max. number of moments/freq.point	16
Initial (POD of RHS) (A1, step 2)	2.155
SF/NF/Solv	4.673 / 6.867 / 0.661
Solution (A2, steps: 1,8)	41.06
Orthogonalization (inside SAPOR) (A2, steps 3,10)	19.33
Local estimator (A2, step 12)	0.163
Global estimator (A1, step 6)	0.242
Update matrix (A1 step 5; A2, steps 4, 11)	4.675
Final frequency sweep	4.02
Number of vectors in the basis	84
Number of expansion freq. points	2
Number of variables	385588
Number of frequency points	100
Lossy	0
Absorbing Boundary conditions	1

2.25 BPF1 (tolerance 1e-4) - 2 modes!



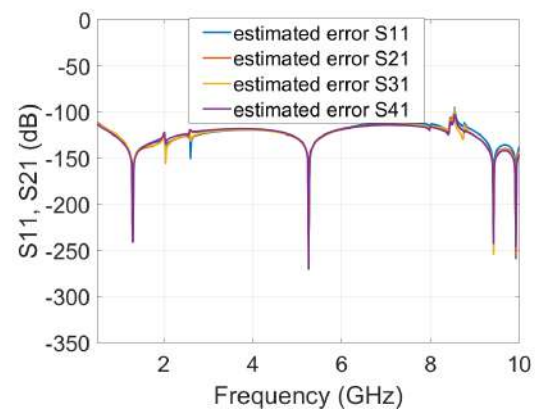
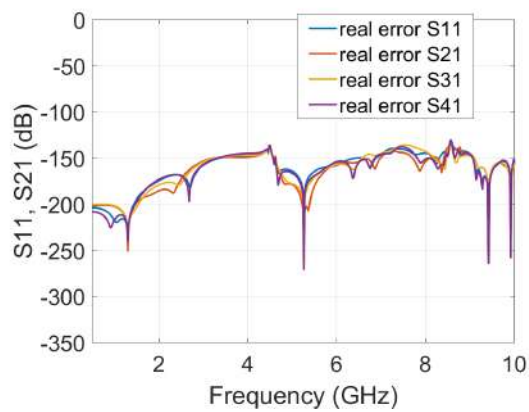
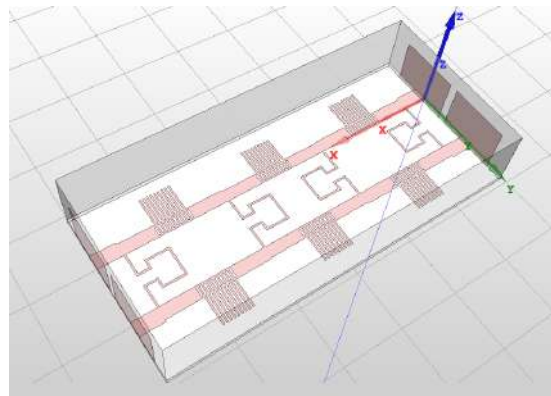
Approx. speed - up	13.244
Reduction time	113.764
Tolerance	0.0001
Max. number of moments/freq.point	16
Initial (POD of RHS) (A1, step 2)	11.68
SF/NF/Solv	5.421 / 6.354 / 0.715
Solution (A2, steps: 1,8)	53.99
Orthogonalization (inside SAPOR) (A2, steps 3,10)	32.65
Local estimator (A2, step 12)	0.264
Global estimator (A1, step 6)	1.114
Update matrix (A1 step 5; A2, steps 4, 11)	6.138
Final frequency sweep	6.175
Number of vectors in the basis	108
Number of expansion freq. points	3
Number of variables	440082
Number of frequency points	201
Lossy	0
Absorbing Boundary conditions	1

2.26 BPF1 (tolerance 1e-6) - 2 modes!



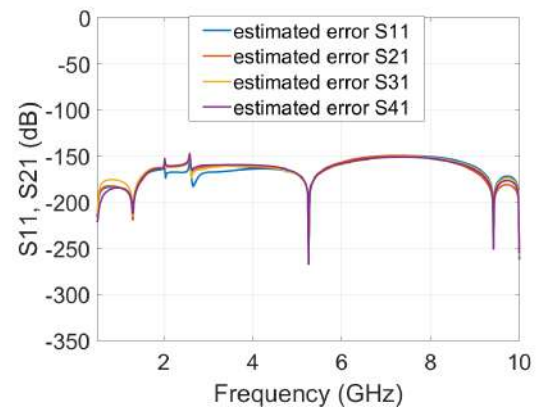
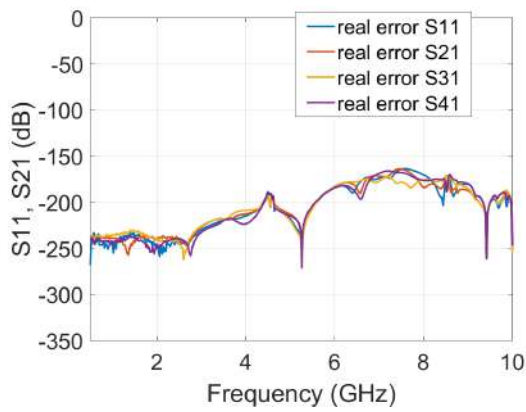
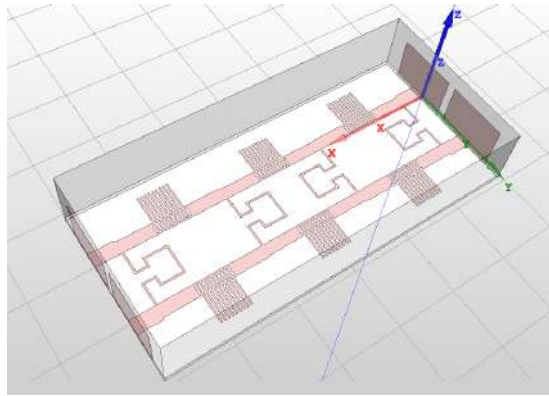
Approx. speed - up	10.536
Reduction time	137.251
Tolerance	1e-006
Max. number of moments/freq.point	16
Initial (POD of RHS) (A1, step 2)	11.57
SF/NF/Solv	5.282 / 6.128 / 0.631
Solution (A2, steps: 1,8)	63.47
Orthogonalization (inside SAPOR) (A2, steps 3,10)	42.83
Local estimator (A2, step 12)	0.386
Global estimator (A1, step 6)	2.248
Update matrix (A1 step 5; A2, steps 4, 11)	7.936
Final frequency sweep	7.471
Number of vectors in the basis	132
Number of expansion freq. points	4
Number of variables	440082
Number of frequency points	201
Lossy	0
Absorbing Boundary conditions	1

2.27 Branch line coupler (tolerance 1e-4)



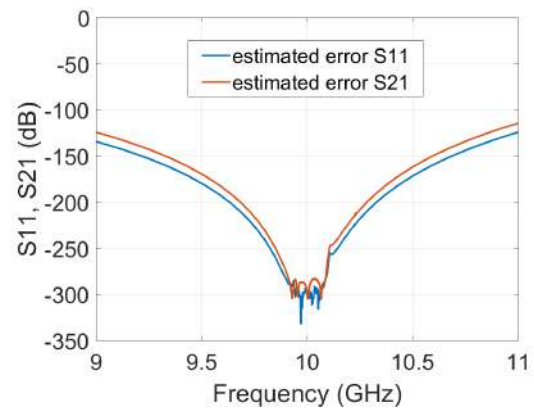
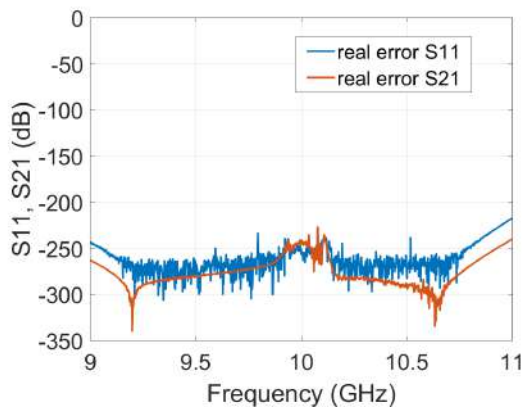
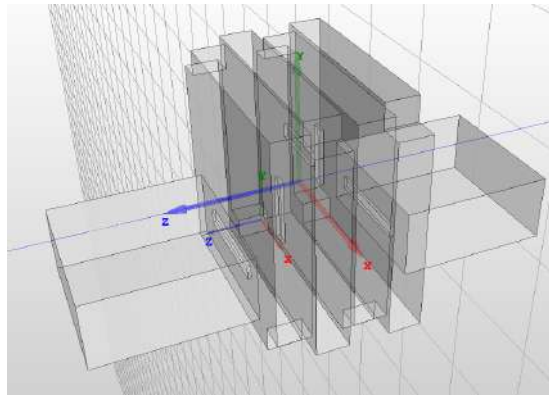
Approx. speed - up	33.514
Reduction time	530.562
Tolerance	0.0001
Max. number of moments/freq.point	16
Initial (POD of RHS) (A1, step 2)	39.03
SF/NF/Solv	17.15 / 36.33 / 3.146
Solution (A2, steps: 1,8)	296
Orthogonalization (inside SAPOR) (A2, steps 3,10)	134.8
Local estimator (A2, step 12)	1.309
Global estimator (A1, step 6)	7.771
Update matrix (A1 step 5; A2, steps 4, 11)	24.94
Final frequency sweep	20.98
Number of vectors in the basis	132
Number of expansion freq. points	4
Number of variables	1153444
Number of frequency points	476
Lossy	0
Absorbing Boundary conditions	1

2.28 Branch line coupler (tolerance 1e-6)



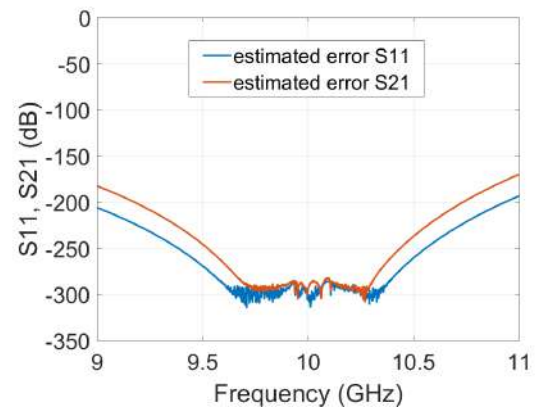
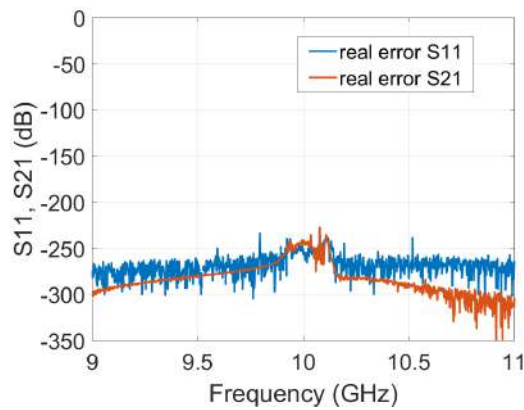
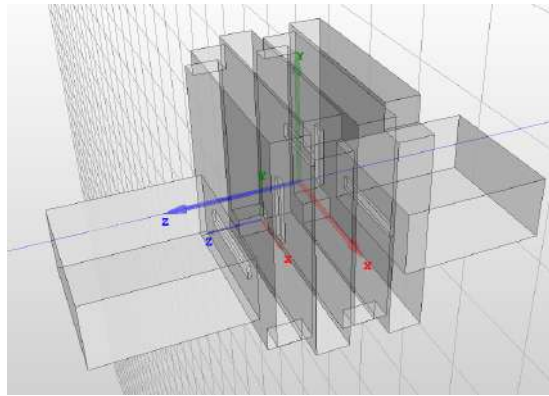
Approx. speed - up	23.734
Reduction time	730.995
Tolerance	1e-006
Max. number of moments/freq.point	16
Initial (POD of RHS) (A1, step 2)	40.57
SF/NF/Solv	17.17 / 31.79 / 2.928
Solution (A2, steps: 1,8)	369.4
Orthogonalization (inside SAPOR) (A2, steps 3,10)	236
Local estimator (A2, step 12)	1.249
Global estimator (A1, step 6)	9.438
Update matrix (A1 step 5; A2, steps 4, 11)	36.39
Final frequency sweep	32.91
Number of vectors in the basis	192
Number of expansion freq. points	5
Number of variables	1153444
Number of frequency points	476
Lossy	0
Absorbing Boundary conditions	1

2.29 Bastioli 2010 (tolerance 1e-4)



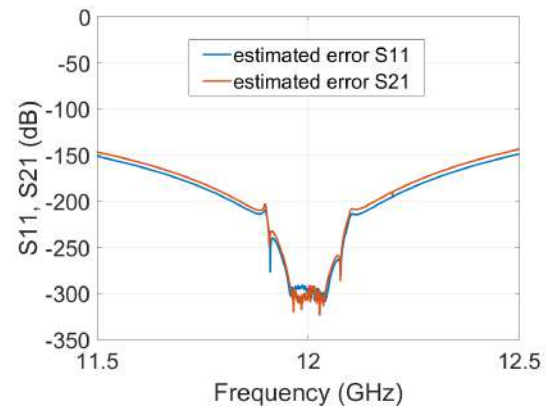
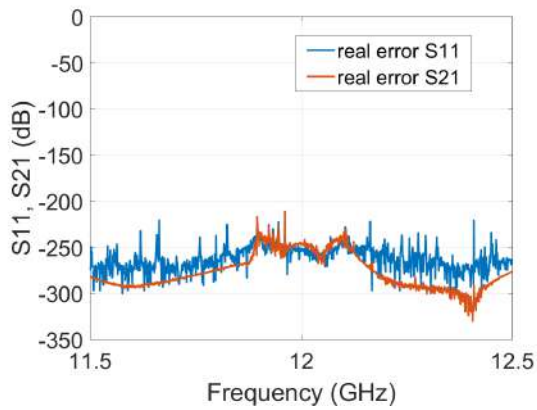
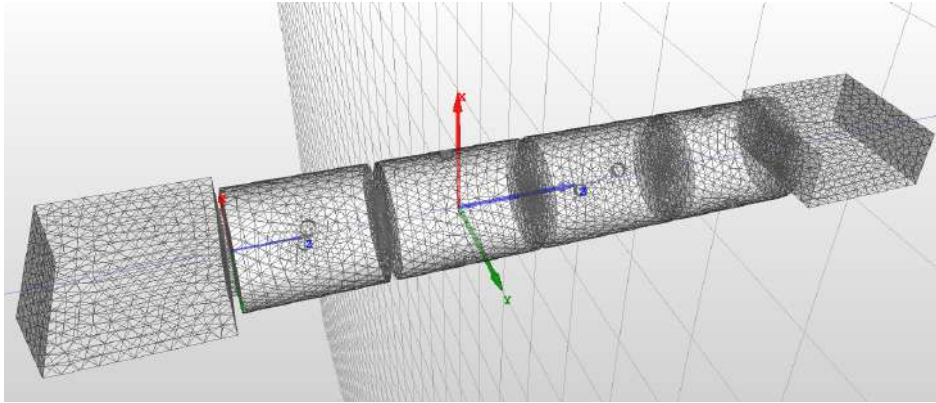
Approx. speed - up	72.925
Reduction time	5.244
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.04
SF/NF/Solv	2.037 / 0.3 / 0.08
Solution (A2, steps: 1,8)	3.771
Orthogonalization (inside SAPOR) (A2, steps 3,10)	0.607
Local estimator (A2, step 12)	0.096
Global estimator (A1, step 6)	0.063
Update matrix (A1 step 5; A2, steps 4, 11)	0.507
Final frequency sweep	0.036
Number of vectors in the basis	24
Number of expansion freq. points	1
Number of variables	170144
Number of frequency points	1001
Lossy	0
Absorbing Boundary conditions	0

2.30 Bastioli 2010 (tolerance 1e-6)



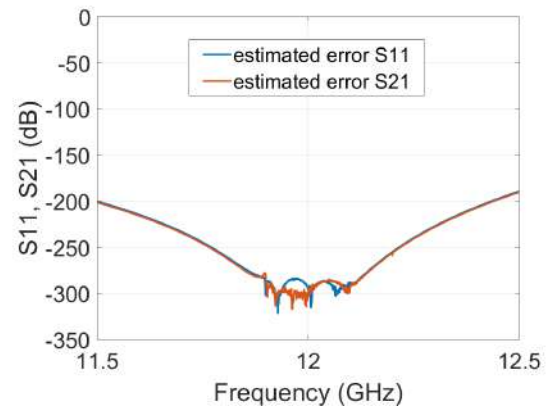
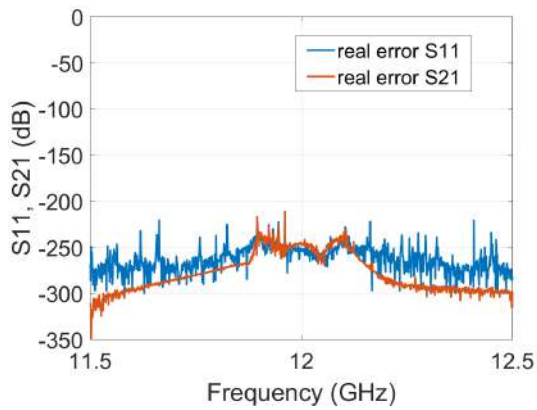
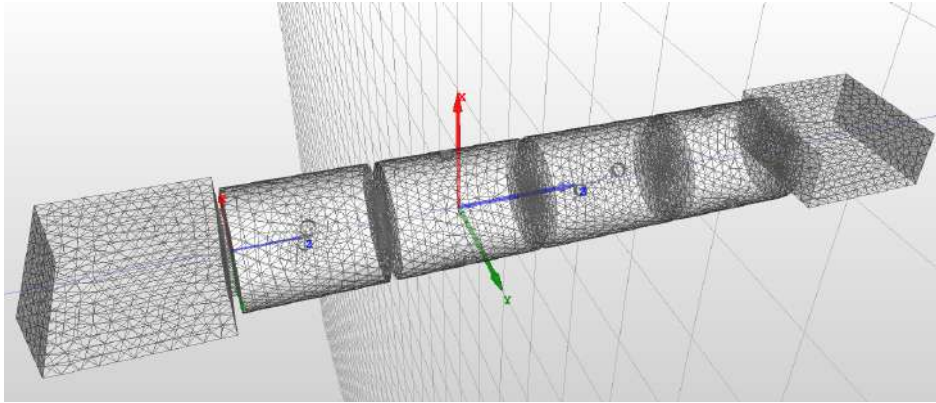
Approx. speed - up	63.313
Reduction time	6.072
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.039
SF/NF/Solv	2.056 / 0.303 / 0.079
Solution (A2, steps: 1,8)	4.1
Orthogonalization (inside SAPOR) (A2, steps 3,10)	0.899
Local estimator (A2, step 12)	0.116
Global estimator (A1, step 6)	0.087
Update matrix (A1 step 5; A2, steps 4, 11)	0.646
Final frequency sweep	0.042
Number of vectors in the basis	30
Number of expansion freq. points	1
Number of variables	170144
Number of frequency points	1001
Lossy	0
Absorbing Boundary conditions	0

2.31 8-th order dual mode (tolerance 1e-4)



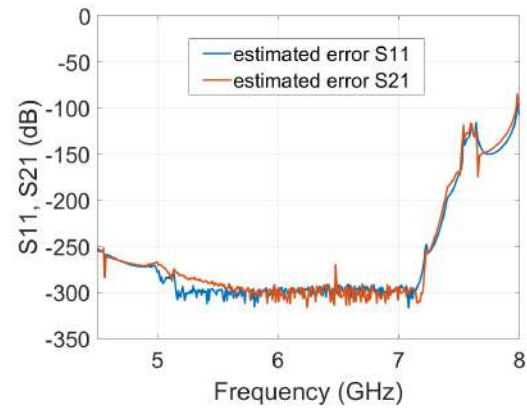
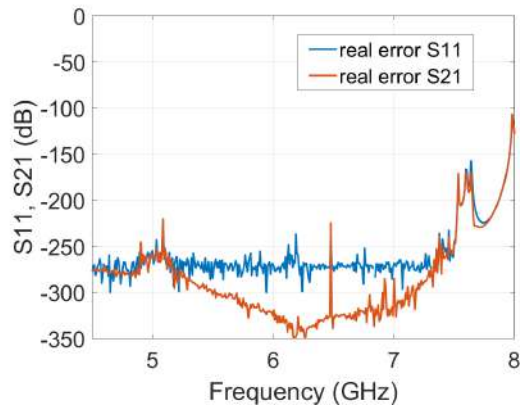
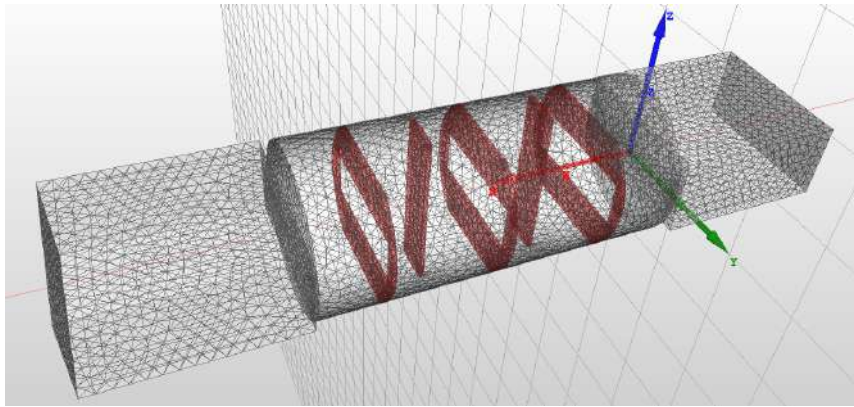
Approx. speed - up	141.236
Reduction time	7.291
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.05
SF/NF/Solv	2.723 / 0.895 / 0.131
Solution (A2, steps: 1,8)	5.565
Orthogonalization (inside SAPOR) (A2, steps 3,10)	0.655
Local estimator (A2, step 12)	0.085
Global estimator (A1, step 6)	0.057
Update matrix (A1 step 5; A2, steps 4, 11)	0.654
Final frequency sweep	0.036
Number of vectors in the basis	20
Number of expansion freq. points	1
Number of variables	239092
Number of frequency points	1001
Lossy	0
Absorbing Boundary conditions	0

2.32 8-th order dual mode (tolerance 1e-6)



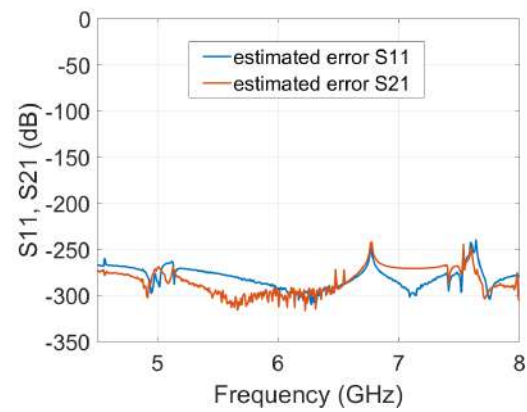
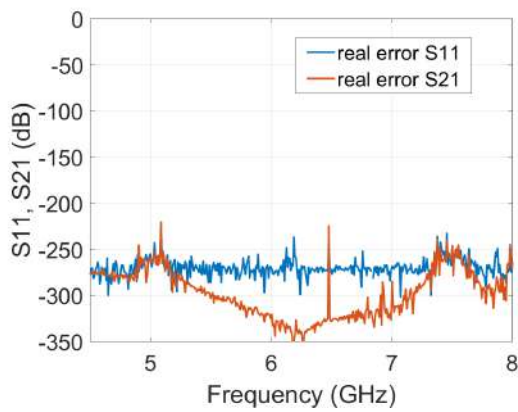
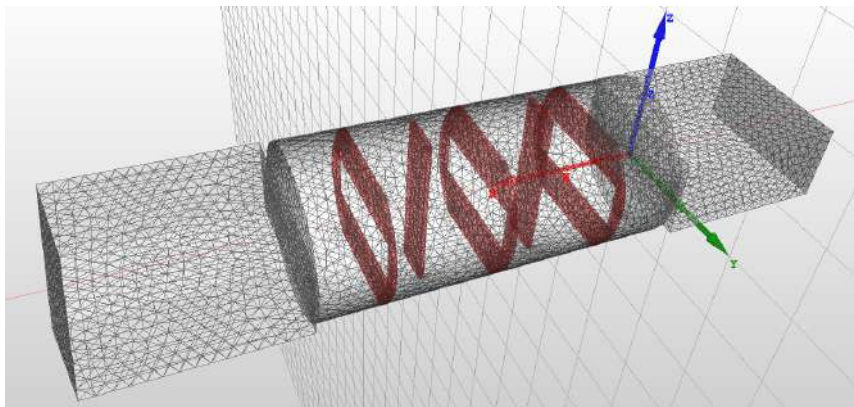
Approx. speed - up	131.099
Reduction time	8.038
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.049
SF/NF/Solv	2.727 / 0.91 / 0.14
Solution (A2, steps: 1,8)	5.947
Orthogonalization (inside SAPOR) (A2, steps 3,10)	0.883
Local estimator (A2, step 12)	0.095
Global estimator (A1, step 6)	0.063
Update matrix (A1 step 5; A2, steps 4, 11)	0.766
Final frequency sweep	0.038
Number of vectors in the basis	24
Number of expansion freq. points	1
Number of variables	239092
Number of frequency points	1001
Lossy	0
Absorbing Boundary conditions	0

2.33 Bila March2014 (tolerance 1e-4)



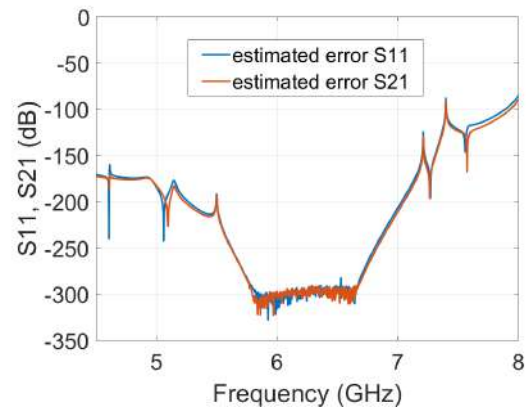
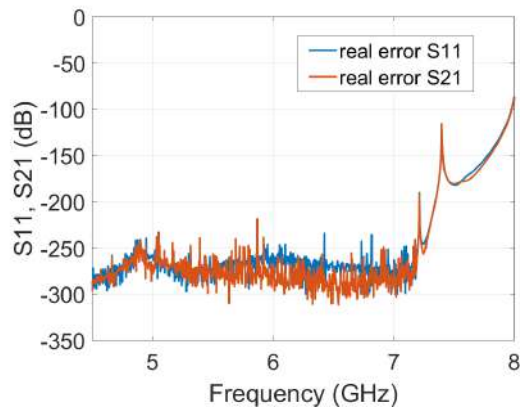
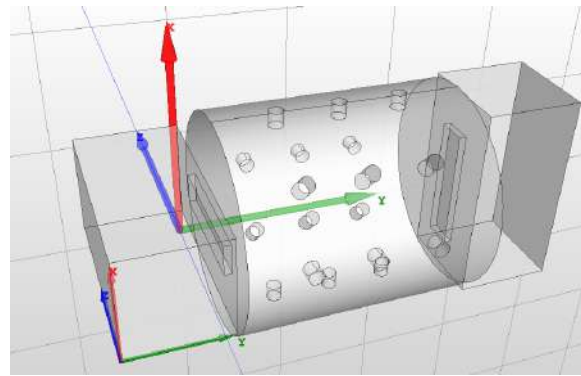
Approx. speed - up	36.293
Reduction time	27.356
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.068
SF/NF/Solv	4.377 / 2.238 / 0.227
Solution (A2, steps: 1,8)	15.02
Orthogonalization (inside SAPOR) (A2, steps 3,10)	6.729
Local estimator (A2, step 12)	0.891
Global estimator (A1, step 6)	0.142
Update matrix (A1 step 5; A2, steps 4, 11)	4.022
Final frequency sweep	0.057
Number of vectors in the basis	54
Number of expansion freq. points	1
Number of variables	363728
Number of frequency points	401
Lossy	0
Absorbing Boundary conditions	0

2.34 Bila March2014 (tolerance 1e-6)



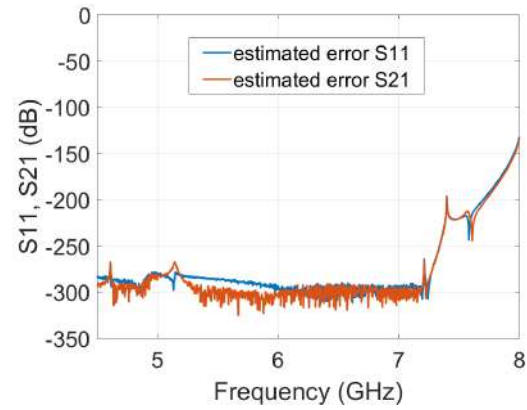
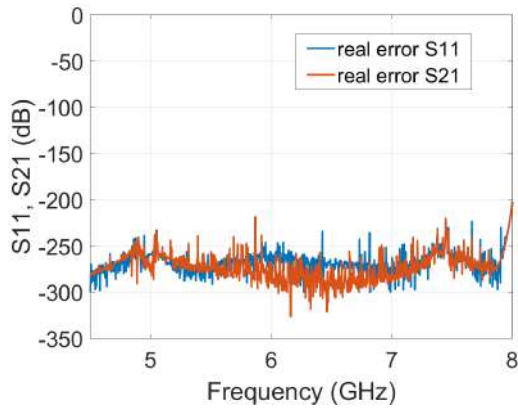
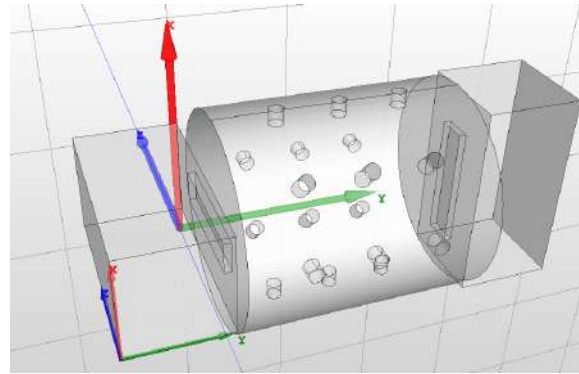
Approx. speed - up	25.784
Reduction time	36.256
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.066
SF/NF/Solv	4.119 / 1.96 / 0.218
Solution (A2, steps: 1,8)	19.4
Orthogonalization (inside SAPOR) (A2, steps 3,10)	11.07
Local estimator (A2, step 12)	0.519
Global estimator (A1, step 6)	0.466
Update matrix (A1 step 5; A2, steps 4, 11)	3.866
Final frequency sweep	0.089
Number of vectors in the basis	74
Number of expansion freq. points	2
Number of variables	363728
Number of frequency points	401
Lossy	0
Absorbing Boundary conditions	0

2.35 Bila August2017 Fig6 (tolerance 1e-4)



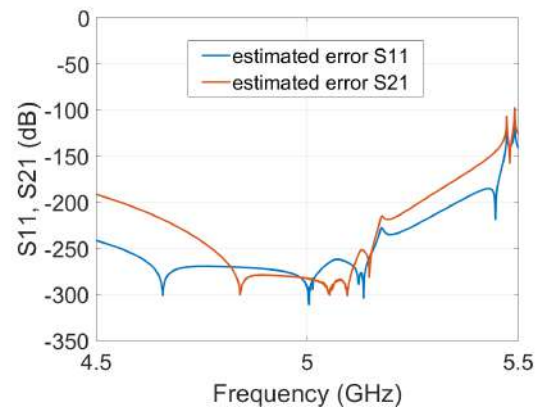
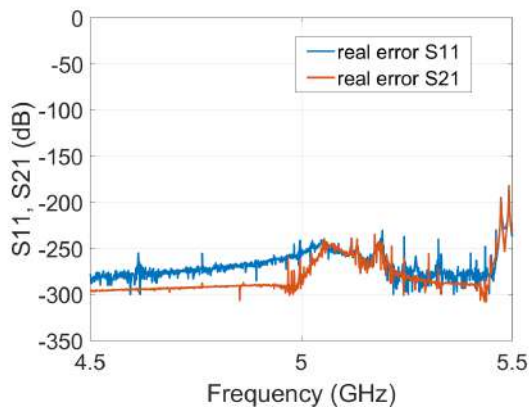
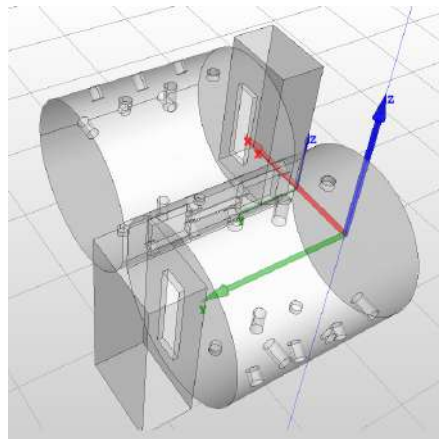
Approx. speed - up	127.172
Reduction time	6.611
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.035
SF/NF/Solv	1.895 / 0.739 / 0.099
Solution (A2, steps: 1,8)	4.514
Orthogonalization (inside SAPOR) (A2, steps 3,10)	0.777
Local estimator (A2, step 12)	0.244
Global estimator (A1, step 6)	0.082
Update matrix (A1 step 5; A2, steps 4, 11)	0.765
Final frequency sweep	0.038
Number of vectors in the basis	28
Number of expansion freq. points	1
Number of variables	163752
Number of frequency points	1001
Lossy	0
Absorbing Boundary conditions	0

2.36 Bila August2017 Fig6 (tolerance 1e-6)



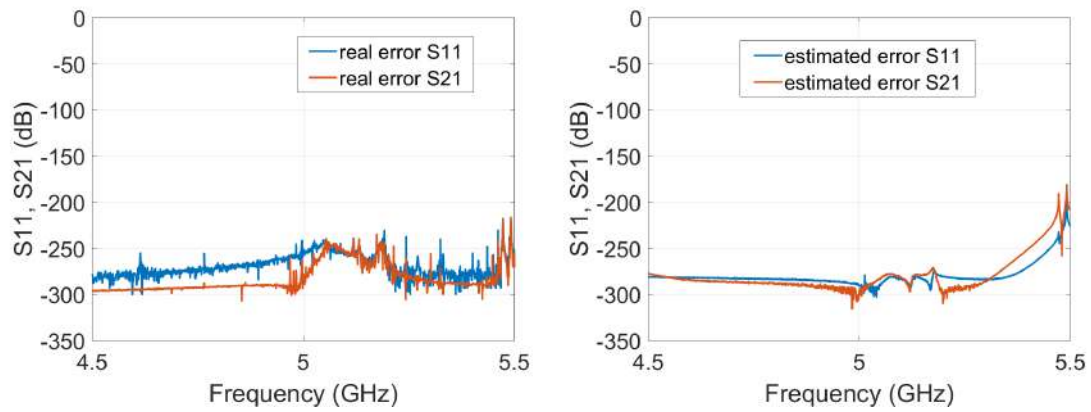
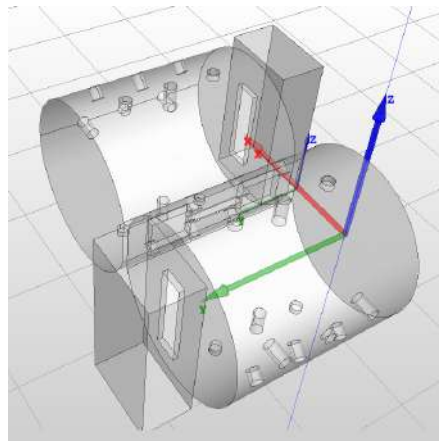
Approx. speed - up	89.676
Reduction time	8.693
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.035
SF/NF/Solv	1.778 / 0.686 / 0.091
Solution (A2, steps: 1,8)	5.081
Orthogonalization (inside SAPOR) (A2, steps 3,10)	1.505
Local estimator (A2, step 12)	0.616
Global estimator (A1, step 6)	0.16
Update matrix (A1 step 5; A2, steps 4, 11)	1.071
Final frequency sweep	0.06
Number of vectors in the basis	42
Number of expansion freq. points	1
Number of variables	163752
Number of frequency points	1001
Lossy	0
Absorbing Boundary conditions	0

2.37 Bila Agust2017 Fig16 (tolerance 1e-4)



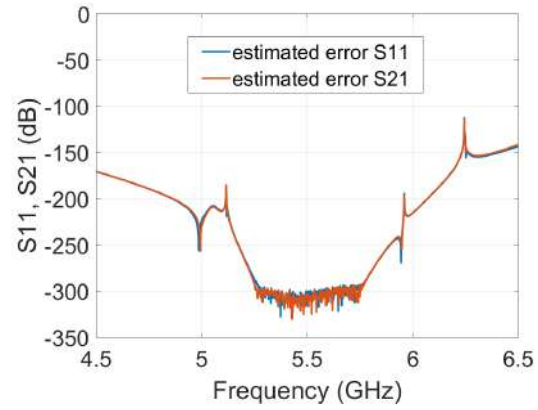
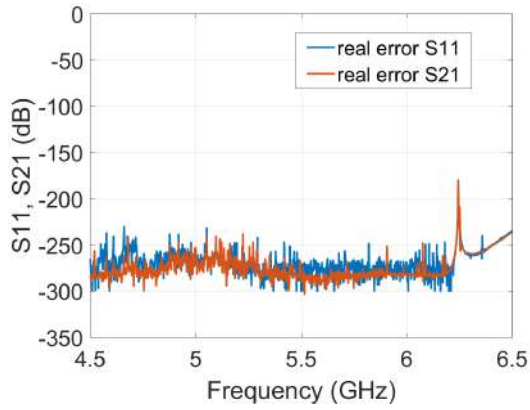
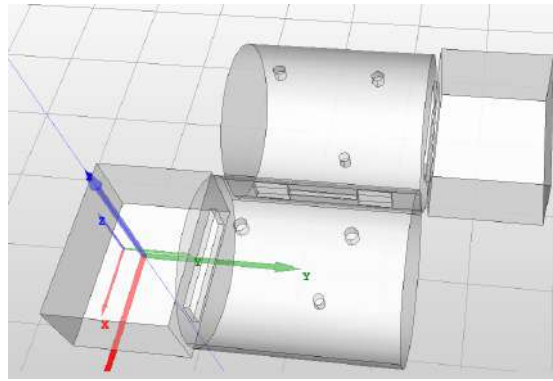
Approx. speed - up	180.844
Reduction time	12.133
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.065
SF/NF/Solv	3.989 / 1.974 / 0.214
Solution (A2, steps: 1,8)	9.298
Orthogonalization (inside SAPOR) (A2, steps 3,10)	1.15
Local estimator (A2, step 12)	0.09
Global estimator (A1, step 6)	0.057
Update matrix (A1 step 5; A2, steps 4, 11)	1.161
Final frequency sweep	0.037
Number of vectors in the basis	22
Number of expansion freq. points	1
Number of variables	334968
Number of frequency points	1001
Lossy	0
Absorbing Boundary conditions	0

2.38 Bila Agust2017 Fig16 (tolerance 1e-6)



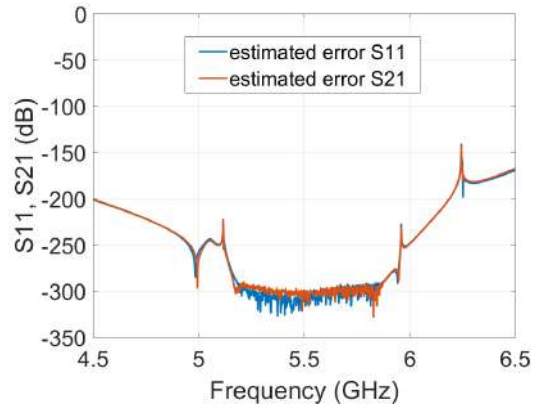
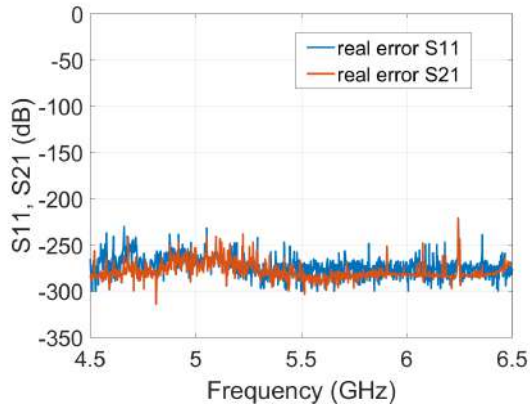
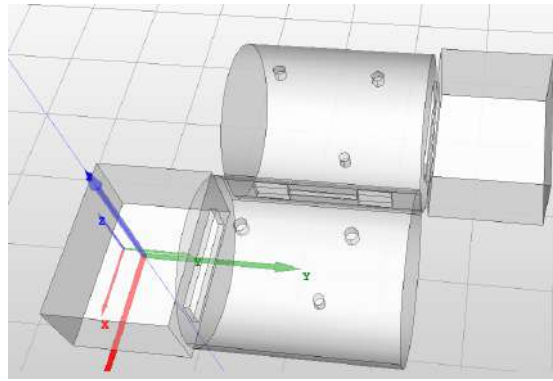
Approx. speed - up	161.424
Reduction time	13.505
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.069
SF/NF/Solv	3.858 / 1.971 / 0.203
Solution (A2, steps: 1,8)	9.835
Orthogonalization (inside SAPOR) (A2, steps 3,10)	1.711
Local estimator (A2, step 12)	0.107
Global estimator (A1, step 6)	0.076
Update matrix (A1 step 5; A2, steps 4, 11)	1.36
Final frequency sweep	0.048
Number of vectors in the basis	28
Number of expansion freq. points	1
Number of variables	334968
Number of frequency points	1001
Lossy	0
Absorbing Boundary conditions	0

2.39 Bila August2017 Fig10 LowBand (tolerance 1e-4)



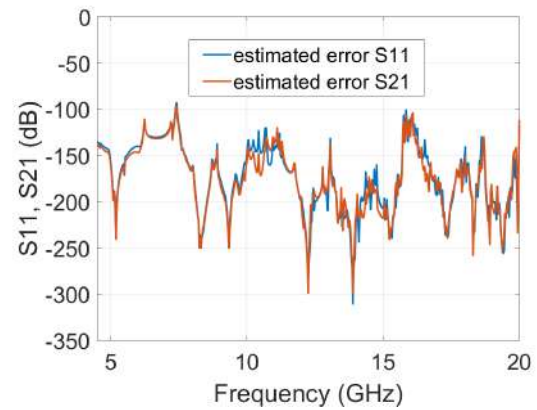
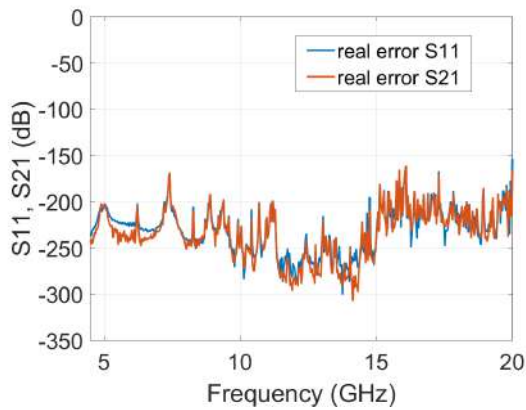
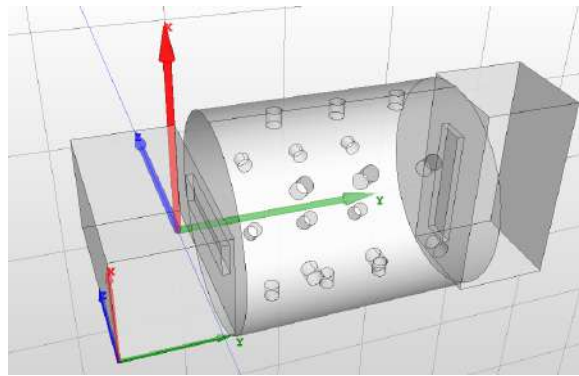
Approx. speed - up	168.613
Reduction time	11.346
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.06
SF/NF/Solv	3.175 / 1.744 / 0.164
Solution (A2, steps: 1,8)	8.195
Orthogonalization (inside SAPOR) (A2, steps 3,10)	1.337
Local estimator (A2, step 12)	0.158
Global estimator (A1, step 6)	0.074
Update matrix (A1 step 5; A2, steps 4, 11)	1.228
Final frequency sweep	0.044
Number of vectors in the basis	28
Number of expansion freq. points	1
Number of variables	274598
Number of frequency points	1001
Lossy	0
Absorbing Boundary conditions	0

2.40 Bila August2017 Fig10 LowBand (tolerance 1e-6)



Approx. speed - up	212.035
Reduction time	15.001
Tolerance	1e-006
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.054
SF/NF/Solv	3.557 / 2.836 / 0.338
Solution (A2, steps: 1,8)	10.88
Orthogonalization (inside SAPOR) (A2, steps 3,10)	1.887
Local estimator (A2, step 12)	0.191
Global estimator (A1, step 6)	0.091
Update matrix (A1 step 5; A2, steps 4, 11)	1.569
Final frequency sweep	0.046
Number of vectors in the basis	30
Number of expansion freq. points	1
Number of variables	274598
Number of frequency points	1001
Lossy	0
Absorbing Boundary conditions	0

2.41 LOSSY BIG Bila August2017 Fig6 (tolerance 1e-4)



Approx. speed - up	24.704
Reduction time	10763.802
Tolerance	0.0001
Max. number of moments/freq.point	32
Initial (POD of RHS) (A1, step 2)	0.371
SF/NF/Solv	34.29 / 548.5 / 6.998
Solution (A2, steps: 1,8)	7525
Orthogonalization (inside SAPOR) (A2, steps 3,10)	2619
Local estimator (A2, step 12)	29.9
Global estimator (A1, step 6)	114.7
Update matrix (A1 step 5; A2, steps 4, 11)	291.6
Final frequency sweep	149.9
Number of vectors in the basis	378
Number of expansion freq. points	11
Number of variables	2187538
Number of frequency points	501
Lossy	1
Absorbing Boundary conditions	0