EDISON

Electromagnetic Design of
flexIble SensOrs

FNP Team-Tech

Report 1 - FEM-MM Formulation for Scattering Electromagnetic Field Computation Enhanced by RBM

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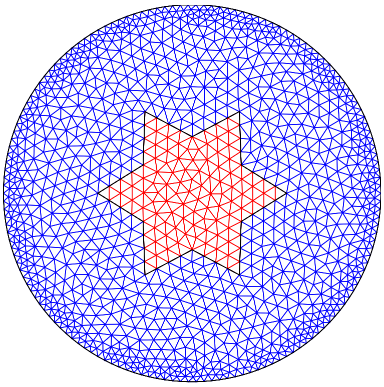
1. Formulation of the problem



Analyzed structure is star. Number of modes at the boundary is defined with the number M, but the final size is $m = -M : M \times 2$. For example if $M = 3$ then $m = 14$. Frequency band is defined with number of frequency points n_f, minimum frequency f_{min} and maximum frequency f_{max}. The original model system of equation size is N and the reduced model size is $n = n_Q \cdot m$ where n_Q is number of frequency points in projection basis.

The goal of this rep is to apply the reduced basis method to scattering problem formulation based on the hybrid FEM and Mode matching method.

1.1. Basis construction

System of equations:	$\mathbf{G}\Psi = -j\omega\mu\mathbf{B}$
System matrix:	$\mathbf{G} \in C^{N\times N}$
Right side vector:	$\mathbf{B} \in C^{N\times m}$
Solution vector:	$\Psi \in C^{N\times m}$
Reduced solution vector:	$\Psi_r \in C^{n\times m}$
Approximated solution vector:	$\Psi \approx \mathbf{Q}\Psi_r$
Local real error:	$\text{norm}(\mathbf{Q}\Psi_r - \Psi)/\text{norm}(\Psi)$

Subsequent projection basis columns are constructed using solution vector Ψ at specific frequencies to represent the evolution of electromagnetic field as a function of frequency. Then SVD algorithm is applied to orthogonalize basis every time it changes. Next frequency points are chosen using error estimator, which is strongly correlated to local real error as shown at numerical experiment section.

1.2. Scattered field

Original field:	$outE$
RBM field:	$outE_r$
Far field error:	$\max(outE - outE_r)$

2. Computational optimization

The block matrix $\mathbf{G}$ is defined as:

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{zz} & \mathbf{G}_{zt} \\ \mathbf{G}_{tz} & \mathbf{G}_{tt} \end{bmatrix}, \tag{1}$$

where inner components depend on frequency variables β and k_0. In order to introduce offline and online phase [1], it is necessary to separate matrices $\mathbf{G}_{zz}$, $\mathbf{G}_{zt}$, $\mathbf{G}_{tz}$, $\mathbf{G}_{tt}$ on frequency independent and frequency dependent components. Model transformation need to be done before combining large matrix $\mathbf{G}$. The projection of matrix $\mathbf{G}$ onto a subspace spanned by $\mathbf{Q}$ vectors is given by:

$$\mathbf{Q}^H\mathbf{G}\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1^H \mathbf{Q}_2^H \end{bmatrix} \begin{bmatrix} \mathbf{G}_{zz} & \mathbf{G}_{zt} \\ \mathbf{G}_{tz} & \mathbf{G}_{tt} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix}. \tag{2}$$

However instead of multiplying matrices in this way, we can take advantage of the fact that matrix $\mathbf{G}$ is constructed with block matrices (1). The result of (2) will be given by:

$$\begin{bmatrix} \mathbf{Q}_1^H \mathbf{Q}_2^H \end{bmatrix} \begin{bmatrix} \mathbf{G}_{zz} & \mathbf{G}_{zt} \\ \mathbf{G}_{tz} & \mathbf{G}_{tt} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix} = \mathbf{Q}_1^H\mathbf{G}_{zz}\mathbf{Q}_1 + \mathbf{Q}_1^H\mathbf{G}_{zt}\mathbf{Q}_2 + \mathbf{Q}_2^H\mathbf{G}_{tz}\mathbf{Q}_1 + \mathbf{Q}_2^H\mathbf{G}_{tt}\mathbf{Q}_2. \tag{3}$$

Taking into account three frequency dependent cases and constructing of block matrices results in:

$$\begin{aligned} \mathbf{G}_{zz}(k_0^2) &= \mathbf{C}_s + k_0^2\mathbf{T}_s, \\ \mathbf{G}_{zt}(\beta) &= \beta\mathbf{G}_{zt}, \\ \mathbf{G}_{tz}(\beta) &= \beta\mathbf{G}_{tz}, \\ \mathbf{G}_{tt}(k_0^2, \beta^2) &= \mathbf{C}_v + k_0^2\mathbf{T}_v + \beta^2\mathbf{G}_v. \end{aligned} \tag{4}$$

Now transformation of matrices should be done using (3):

$$\begin{aligned} \mathbf{C}_{rs} &= \mathbf{Q}_1^H\mathbf{C}_s\mathbf{Q}_1, & & & \mathbf{T}_{rs} &= \mathbf{Q}_1^H\mathbf{T}_s\mathbf{Q}_1, \\ \mathbf{C}_{rv} &= \mathbf{Q}_2^H\mathbf{C}_v\mathbf{Q}_2, & \mathbf{T}_{rv} &= \mathbf{Q}_2^H\mathbf{T}_v\mathbf{Q}_2, & \mathbf{G}_{rv} &= \mathbf{Q}_2^H\mathbf{G}_v\mathbf{Q}_2, \\ \mathbf{G}_{rzt} &= \mathbf{Q}_2^H\mathbf{G}_{tz}\mathbf{Q}_1, & & & \mathbf{G}_{rtz} &= \mathbf{Q}_1^H\mathbf{G}_{zt}\mathbf{Q}_2. \end{aligned} \tag{5}$$

Combining (4) and (5) allows us to separate block sub-matrices in following way:

$$\begin{aligned} \mathbf{G}_0 =& \mathbf{C}_{rs} + \mathbf{C}_{rv}, \\ \mathbf{G}_\beta =& \mathbf{G}_{rz} + \mathbf{G}_{rtz}, \\ \mathbf{G}_{\beta^2} =& \mathbf{G}_{rv}, \\ \mathbf{G}_{k_0^2} =& \mathbf{T}_{rs} + \mathbf{T}_{rv}. \end{aligned} \tag{6}$$

At this moment we obtained frequency independent matrices with the same low order $n \times n$. Indexes of matrices correspond to frequency relations. The reduced frequency depended matrix $\mathbf{G}(f)$ is now obtained with following formula using (6):

$$\mathbf{G}_r(f) = \mathbf{G}_0 + \beta\mathbf{G}_\beta + \beta^2\mathbf{G}_{\beta^2} + k_0^2\mathbf{G}_{k_0^2}. \tag{7}$$

The error estimator is given by:

$$\begin{aligned} \mathbf{e}(f) &= \mathrm{norm}(\mathbf{B}^H/\mathrm{norm}(\mathbf{B}) \cdot (\mathbf{G}\mathbf{Q}\Psi + j\omega\mu\mathbf{B})/\mathrm{norm}(j\omega\mu\mathbf{B})) \\ \mathbf{e}(f) &= \mathrm{norm}(\mathbf{B}^H\mathbf{G}\mathbf{Q}\Psi + j\omega\mu\mathbf{B}^H\mathbf{B})/\mathrm{norm}(j\omega\mu)/\mathrm{norm}(\mathbf{B})^2) \end{aligned} \tag{8}$$

Note that $\mathbf{B}$ is frequency independent, so $\mathbf{B}^H\mathbf{G}\mathbf{Q}$, $\mathbf{B}^H\mathbf{B}$ and $\mathrm{norm}(\mathbf{B})$ can be computed before frequency sweep, which makes this estimator very fast operation on low order $n \times n$ matrices.

3. Numerical Experiments

Experiments were performed with three input variables n_f, M and tol. Tests does not include mesh and matrix generation.

Table 1: Analysis result for $N =$**15729**, $tol = 1e-4$, $n_f = 81$

M	n_Q	FEM time	RBM time	Speedup	MaxEst
3	10	44.55	16.8	2.65	1.26e-5
5	8	65.49	19.6	3.34	6.37e-5
10	7	114.1	35.9	3.18	4.17e-5
15	8	161.6	63.9	2.53	1.25e-5

Table 2: Analysis result for $N =$**44247**, $tol = 1e-4$, $n_f = 81$

M	n_Q	FEM time	RBM time	Speedup	MaxEst
3	17	151.8	76.3	1.98	1.09e-5
5	16	218.9	109.9	1.99	4.28e-6
10	12	373.8	148.9	2.51	4.82e-6
15	10	535.4	176.6	3.03	1.25e-5

Subsequent plots present normalized far field error, solution vector Ψ error which is local real error, error estimator and far field scattering distribution at maximum far field error frequency point. Black circles marks the global maxima of following functions. The three dimensional charts represents far field scattering error depended on Φ angle, frequency and basis size.

References

[1] De La Rubia, Valentin, Ulrich Razafison, and Yvon Maday. *Reliable fast frequency sweep for microwave devices via the reduced-basis method.* IEEE Transactions on Microwave Theory and Techniques 57.12 (2009): 2923-2937.