EDISON

Electromagnetic Design of
flexIble SensOrs

FNP Team-Tech

Report 2 - FEM-MM Formulation for Scattering Electromagnetic Field Computation Enhanced by MOR

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July 6, 2018

European Funds
Smart Growth

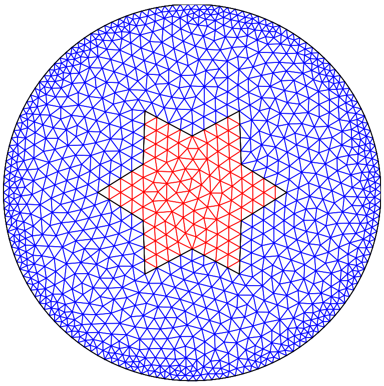
FNP Foundation for Polish Science

European Union
European Regional
Development Fund

The „EDISOn - Electromagnetic Design of flexIble SensOrs" project, agreement no TEAM TECH/2016-1/6, is carried out within the TEAM-TECH programme of the Foundation for Polish Science co-financed by the European Union under the European Regional Development Fund.

Revision	Date	Author(s)	Description
1.0	22.08.2017		created

1. Formulation of the problem



Analyzed structure is star. Number of modes at the boundary is defined with the number M, but the final size is $m = -M : M \times 2$. For example if $M = 3$ then $m = 14$. Frequency band is defined with number of frequency points n_f, minimum frequency f_{min} and maximum frequency f_{max}. The original model system of equation size is N and the reduced model size is $n = n_Q \cdot m$ where n_Q is number of subsequent block moments at subsequent expansion points.

The goal of this rep is to compare RBM, SAPOR and GM-MOR algorithms application to scattering problem formulation based on the hybrid FEM and Mode matching method.

1.1. Problem construction

System of equations:	$\mathbf{G}\Psi = -j\omega\mu\mathbf{B}$
System matrix:	$\mathbf{G} \in C^{N\times N}$
Right side vector:	$\mathbf{B} \in C^{N\times m}$
Solution vector:	$\Psi \in C^{N\times m}$
Reduced solution vector:	$\Psi_r \in C^{n\times m}$
Approximated solution vector:	$\Psi \approx \mathbf{Q}\Psi_r$
Local real error:	$\mathrm{norm}(\mathbf{Q}\Psi_r - \Psi)/\mathrm{norm}(\Psi)$

Subsequent projection basis columns are constructed using block moments of Ψ expansion point at specific frequencies to represent the evolution of electromagnetic field as a function of frequency. Next expansion points are chosen using error estimator.

1.2. Scattered field

Original field:	$outE$
MOR field:	$outE_r$
Far field error:	$\max(outE - outE_r)$

2. Computational optimization

The block matrix $\mathbf{G}$ is defined as:

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{zz} & \mathbf{G}_{zt} \\ \mathbf{G}_{tz} & \mathbf{G}_{tt} \end{bmatrix}, \tag{1}$$

where inner components depend on frequency variables $k_0 = 2\pi f$ and $\beta = jk_0 cos(\theta)$, where θ is a angle of falling wave. Taking into account three frequency dependent cases and constructing of block matrices results in:

$$\begin{aligned} \mathbf{G}_{zz}(k_0^2) &= \mathbf{C}_s + k_0^2 \mathbf{T}_s, \\ \mathbf{G}_{zt}(\beta) &= \beta \mathbf{G}_{zt}, \\ \mathbf{G}_{tz}(\beta) &= \beta \mathbf{G}_{tz}, \\ \mathbf{G}_{tt}(k_0^2, \beta^2) &= \mathbf{C}_v + k_0^2 \mathbf{T}_v + \beta^2 \mathbf{G}_v. \end{aligned} \tag{2}$$

Consider $cos(\theta)$ as constant and introduce $s = j\omega/c$. After that we can introduce $cos\theta$ as a component of system matrices to obtain following formulation of problem:

$$\begin{aligned} \mathbf{G}\boldsymbol{\Psi} &= -j\omega\mu\mathbf{B}, \\ \mathbf{G} &= s^2\mathbf{T} + s\mathbf{G}_{temp} + \mathbf{C}. \end{aligned} \tag{3}$$

The construction of matrices $\mathbf{C}$, $\mathbf{T}$ and $\mathbf{G}_{temp}$ is organized as follows:

$$\begin{aligned} \mathbf{C} &= \begin{bmatrix} \mathbf{C}_s & 0 \\ 0 & \mathbf{C}_v \end{bmatrix}, \\ \mathbf{T} &= \begin{bmatrix} \mathbf{T}_s & 0 \\ 0 & \mathbf{T}_v + \mathbf{G}_v \cdot cos^2(\theta) \end{bmatrix}, \\ \mathbf{G}_{temp} &= \begin{bmatrix} 0 & \mathbf{G}_{zt} \cdot cos(\theta) \\ \mathbf{G}_{tz} \cdot cos(\theta) & 0 \end{bmatrix}. \end{aligned} \tag{4}$$

At this moment we obtained frequency independent matrices with the same order $N \times N$. We can now introduce error estimator given by:

$$\begin{aligned} \mathbf{e}(f) &= \text{norm}(\mathbf{B}^H/\text{norm}(\mathbf{B}) \cdot (\mathbf{G}\mathbf{Q}\Psi + j\omega\mu\mathbf{B})/\text{norm}(j\omega\mu\mathbf{B})) \\ \mathbf{e}(f) &= \text{norm}(\mathbf{B}^H\mathbf{G}\mathbf{Q}\Psi + j\omega\mu\mathbf{B}^H\mathbf{B})/\text{norm}(j\omega\mu)/\text{norm}(\mathbf{B})^2) \end{aligned} \tag{5}$$

Note that $\mathbf{B}$ is frequency independent, so $\mathbf{B}^H\mathbf{C}\mathbf{Q}$, $\mathbf{B}^H\mathbf{T}\mathbf{Q}$, $\mathbf{B}^H\mathbf{G}_{temp}\mathbf{Q}$, $\mathbf{B}^H\mathbf{B}$ and norm($\mathbf{B}$) can be computed before frequency sweep, which makes this estimator very fast operation on low order $n \times n$ matrices. To evaluate the action of proposed error estimator, we have introduced the real local error given by:

$$\text{norm}(|\mathbf{Q}\Psi_r - \Psi_{ref}|)/\text{norm}(\Psi_{ref}).$$

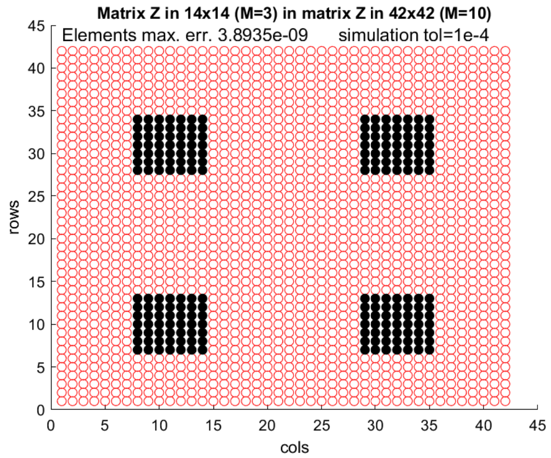
We have obtained good correlation between estimator and local real error. The results will be presented in next sections. Additionally we have introduced impedance matrix error estimator:

$$\text{norm}(|\mathbf{Z}_r - \mathbf{Z}_{ref}|)/\text{norm}(\mathbf{Z}_{ref}),$$

but similarly as in local real error, the reference is not available at the moment of random structure computation. This comparisons may be useful at scientific research but not at industrial applications.

3. Impedance matrix and solution vector Ψ observation

Impedance matrix and solution vector Ψ have the same core elements for M=5 at M=6. It means that removing outer and inner columns of M=6, results in obtaining M=5. We can observe this occurence at fig. 1. Additionally the element error relies on tolerance of simulation. The Z matrix error is lower in contrary to Ψ error, which can be seen at report attachment.



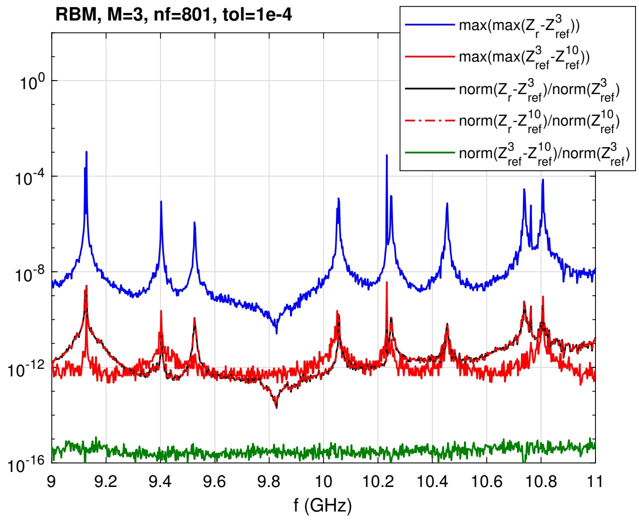


Figure 1: On the left: Black elements are elements obtained via RBM case and red circles are obtained via reference. Comparison made at frequency vector present in projection basis.
On the right: Comparison of Z matrix elements. Comparison is made with reference matrices with M=3 and M=10 obtained via RBM at M=3.

4. Numerical Experiments

Experiments were performed with three input variables number of frequency subranges NF, number of Hankel functions M and maximum number of block moments Q_{max}. Tests does not include mesh and matrix generation. All tests were performed for 9-11 GHz band at $n_f = 801$ points and $tol = 1e-4$.

Table 1: Analysis result for M =**3**

	n	Full time	MOR time	Sweep Time	Speedup	Max. Est.
Original formulation	15 729	443 s	0 s	443 s	1	-
RBM	140	14.7 s	6.94 s	7.76 s	30.1	7.71e-6
SAPOR	182	22.2	14.24 s	7.96 s	19.9	6.93e-7
GM-MOR						
$NF = 10$, $Q_{max} = 10$	154	18.5 s	10.7 s	7.8 s	23.9	1.03e-6
$NF = 20$, $Q_{max} = 5$	154	19 s	11.2 s	7.8 s	23.3	1.91e-6
$NF = 40$, $Q_{max} = 2$	154	18.9 s	11.1 s	7.8 s	23.4	1.86e-6

Table 2: Analysis result for M =**5**

	n	Full time	MOR time	Sweep Time	Speedup	Max. Est.
Original formulation	15 729	612 s	0 s	612 s	1	-
RBM	220	22.7 s	11.3 s	11.4 s	26.9	1.17e-7
SAPOR	242	32.3	20.8 s	11.5 s	18.9	2.18e-5
GM-MOR						
$NF = 10$, $Q_{max} = 10$	198	26.8	15.6	11.3	22.8	6.50e-6
$NF = 20$, $Q_{max} = 5$	198	30.2	18.8	14.4	20.3	2.51e-6
$NF = 40$, $Q_{max} = 2$	198	27.6	16.4	11.2	22.2	5.61e-6

Table 3: Analysis result for M =**10**

	n	Full time	MOR time	Sweep Time	Speedup	Max. Est.
Original formulation	15 729	1120 s	0 s	1120 s	1	-
RBM	378	55.2 s	25.6 s	29.6 s	20.3	2.14e-5
SAPOR	378	77.4 s	47.6 s	29.8 s	14.5	4.25e-5
GM-MOR						
$NF = 10$, $Q_{max} = 10$	336	67.2	38.5	28.7	16.7	3.09e-5
$NF = 20$, $Q_{max} = 5$	378	84.1	54.5	29.6	13.3	7.39e-7
$NF = 40$, $Q_{max} = 2$	378	84.7	54.9	29.8	13.2	1.10e-5

References

[1] Fotyga, Grzegorz, et al. *Reliable Greedy Multipoint Model-Order Reduction Techniques for Finite-Element Analysis.* IEEE Antennas and Wireless Propagation Letters 17.5 (2018): 821-824.

[2] De La Rubia, Valentin, Ulrich Razafison, and Yvon Maday. *Reliable fast frequency sweep for microwave devices via the reduced-basis method.* IEEE Transactions on Microwave Theory and Techniques 57.12 (2009): 2923-2937.