Electromagnetic Design of
flexIble SensOrs

Report 96 CISS for zeros-poles, error

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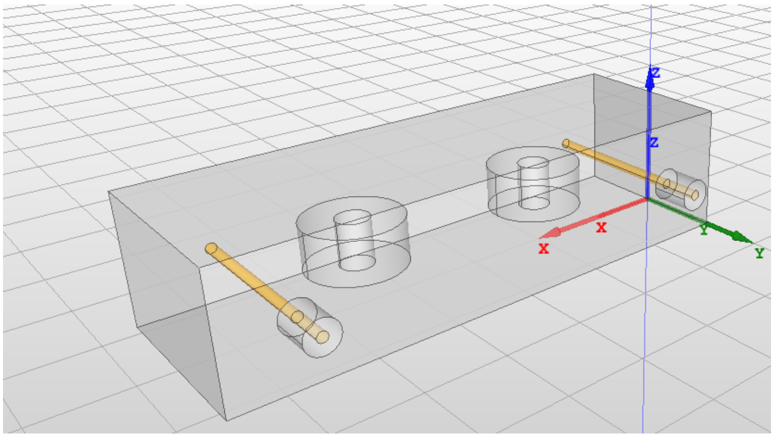

Figure 1: Bandpass filter.

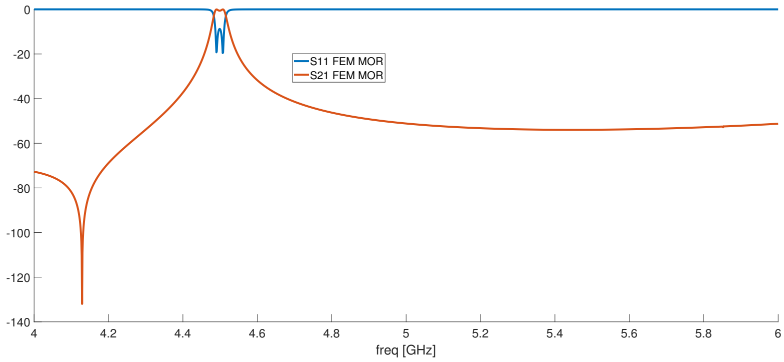

Figure 2: S - characteristics (ROM).

1 Test Structure

- Bandpass filter.
- Poles and zeros are extracted from the eigenproblem associated with reduced order model.

2 Eigenproblem

Model order reduction system of equations (S-params formulations):

$$\begin{cases} (\mathbf{T}_r - k_0^2\mathbf{C}_r + ik_0c\epsilon_0(\mathbf{B}_r d)\cdot(\mathbf{B}_r d)^T)\cdot\mathbf{x}_r = 2\mathbf{b}_r^j da \\ b = ik_0c\epsilon_0(\mathbf{b}_r^i d)^T\mathbf{x}_r - a \end{cases} \tag{1}$$

which can be transformed to the matrix form (assuming $b = 0$):

$$\left(\begin{bmatrix}\mathbf{T}_r & -2\mathbf{b}_r^j d \\ 0 & -\delta_{ij}\end{bmatrix} + k_0\begin{bmatrix} ic\epsilon_0(\mathbf{B}_r d)\cdot(\mathbf{B}_r d)^T & 0 \\ ic\epsilon_0(\mathbf{b}_r^i d)^T & 0\end{bmatrix} + k_0^2\begin{bmatrix}-\mathbf{C}_r & 0 \\ 0 & 0\end{bmatrix}\right)\begin{bmatrix}\mathbf{x}_r \\ a\end{bmatrix} = \begin{bmatrix}0 \\ 0\end{bmatrix}$$

The above eigenproblem is solved using CISS in order to compute **transmission and reflection zeros** of the scattering parameters. **Poles** are computed by solving the eigenproblem:

$$(\mathbf{T}_r - k_0^2\mathbf{C}_r + ik_0c\epsilon_0(\mathbf{B}_r d)\cdot(\mathbf{B}_r d)^T)\cdot\mathbf{x}_r = 0 \tag{2}$$

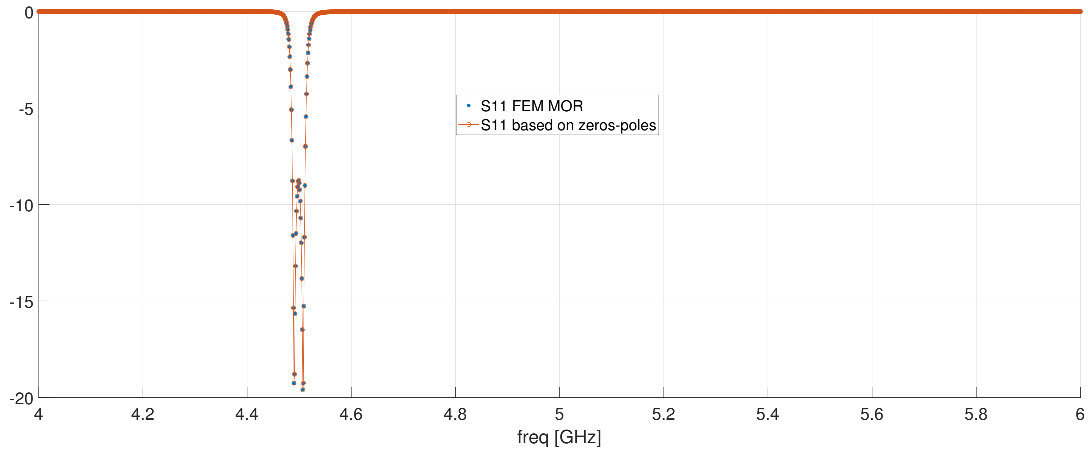

Figure 3: S11 - characteristics (ROM/zeros-poles from band: 3-10GHz).

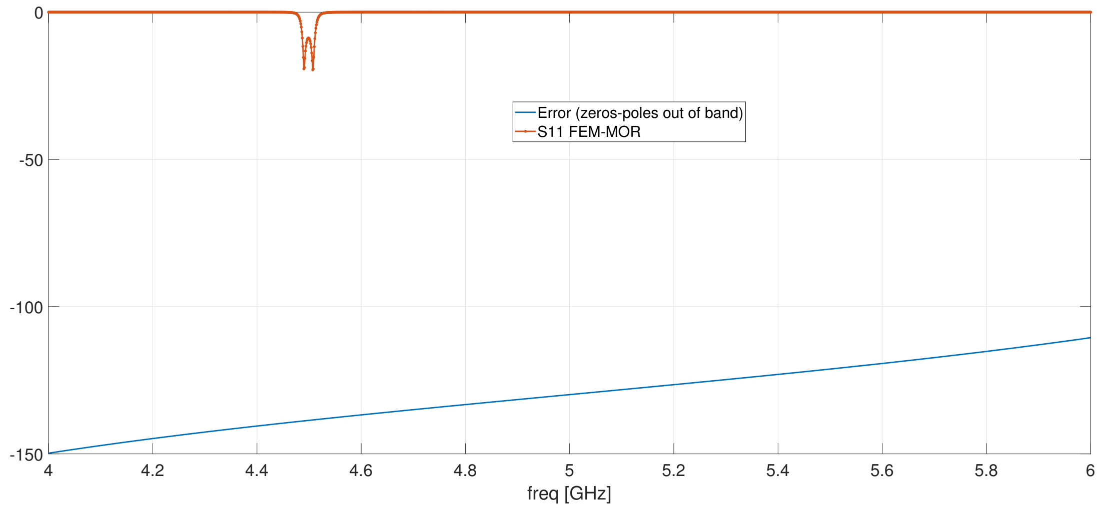

Figure 4: S11 - characteristic obtained using ROM and the error ϵ computed using (3).

3 S-characteristics obtained using zeros-poles compared to the original ones

The plot below (Fig.3) shows $S11$ characteristic based on the FEM MOR approach and characteristic S_{11}^{ZP} obtained using zeros-poles from the band 3-10GHz:

$$S_{11}^{ZP} = 20log_{10}\left(\left|\frac{(s-z_1)\cdot(s-z_2)\cdot\ldots\cdot(s-z_n)}{(s-p_1)\cdot(s-p_2)\cdot\ldots\cdot(s-p_m)}\right|\right) \tag{3}$$

- $s = j\omega$
- n is the numeber of zeros in the band $3-10GHz$
- m is the numeber of poles in the band $3-10GHz$
- z are zeres from band $3-10GHz$
- p are poles from band $3-10GHz$

4 Error

The next plot (Fig.4) show the same S11 characteristic together with the error ϵ:

$$\epsilon = 20log_{10}\left(\left|\frac{(s-z_1)\cdot(s-z_2)\cdot...\cdot(s-z_n)}{(s-p_1)\cdot(s-p_2)\cdot...\cdot(s-p_m)}\right| - \left|\frac{(s-\tilde{z}_1)\cdot(s-\tilde{z}_2)\cdot...\cdot(s-\tilde{z}_k)}{(s-\tilde{p}_1)\cdot(s-\tilde{p}_2)\cdot...\cdot(s-\tilde{p}_l)}\right|\right) \quad (4)$$

where:

- $s = j\omega$
- n is the numeber of zeros in the band $3 - 10GHz$
- m is the numeber of poles in the band $3 - 10GHz$
- k is the numeber of zeros in the band $4 - 6GHz$
- l is the numeber of poles in the band $4 - 6GHz$
- z and $\tilde{z}$ are zeres from band $3 - 10GHz$ and sub-band $4 - 6GHz$, respectively
- p and $\tilde{p}$ are poles from band $3 - 10GHz$ and sub-band $4 - 6GHz$, respectively