

EDISON

Electromagnetic Design of
flexIble SensOrs



Report 15 - Eigs, JDQZ and LOBPCG study

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1. Basic information

The aim of this report is to speed-up solution of generalised eigenproblem for Finite Element Method microwave models, using JDQZ and LOBPCG in comparison to Matlab eigs.

Additional option in JDQZ and LOBPCG is that we can define starting space (matrix), which we assume can speed-up computations.

The plan is as follows:

1. Combine MKL Pardiso with JDQZ
2. Verify eigenvalues with eigs
3. Create starting space for JDQZ
 - Solve FEM equations at middle frequency point $(\mathbf{C} + \mathbf{s}^2\mathbf{\Gamma})\mathbf{E} = \mathbf{B}$
 - Create projection basis from obtained solution point $\mathbf{Q} = \text{svd}(\mathbf{E})$
 - Perform Galerkin projection $\mathbf{C}_R = \mathbf{Q}^T \cdot \mathbf{C}\mathbf{Q}$, $\mathbf{\Gamma}_R = \mathbf{Q}^T \cdot \mathbf{\Gamma}\mathbf{Q}$
 - Solve generalised eigenproblem of reduced order matrices $[\mathbf{V}_R, \Lambda_R] = \text{eigs}(\mathbf{\Gamma}_R, \mathbf{C}_R)$
 - Perform backward projection (ROM on FOM) of obtained eigenvectors: $\mathbf{W}_{\text{start}} = \mathbf{Q} \cdot \mathbf{V}_R$
4. Use obtained vector $\mathbf{W}_{\text{start}}$ as a starting space for JDQZ

Two structures are taken under consideration. The first is 2-port DualMode filter and the second is 32-port PCB Line model.

1.1. Nullspace filtration

In case of LOBPCG algorithm, nullspace filtration is necessary. Otherwise eigenvalues close to 0 are returned.

2. Results

2.1. Two-port 412k variables - dual mode filter

2.1.1. Default starting space

As can be seen in the Tables 2 and 3, the fastest algorithm to compute lower number of eigenvalues (in this case 15) is JDQZ. However, as the number of requested eigenvalues increases, eigs becomes the 1st choice algorithm, and JDQZ is the slowest one. In terms of number of solves (MKL PARDISO 3rd phase), eigs is the most efficient algorithm, and LOBPCG is the most expensive, however it does not affect total time of simulation as can be seen in Table 2.

What is important, the LOBPCG algorithm is not repeatable, this means that in few cases simulation converges, but sometimes it doesn't (algorithm reaches 300 iterations).

What should be noted is that, the Tolerance in case of eigs is lowered (default it is 1e-14).

| Method | Time | No. Solves |
|--------|------|------------|
| Eigs | 35.7 | 55 |
| LOBPCG | 37.3 | 417 |
| JDQZ | 27.5 | 75 |

Table 2: Time comparison for $N_{ev} = 15$ for Dual-Mode filter.

| Method | Time | No. Solves | Tolerance | MaxIterations | Repeatable |
|--------|-------|------------|-----------|---------------|------------|
| Eigs | 60.6 | 107 | 1e-10 | - | Yes |
| LOBPCG | 185.5 | 1812 | 1e-4 | 300 | No |
| JDQZ | 252.8 | 309 | 1e-4 | - | Yes |

Table 3: Time comparison for $N_{ev} = 35$ for Dual-Mode filter.

where N_{ev} is number of eigenvalues to be computed.

The Figs.1-2 show the convergence of JDQZ for $N_{ev} = 15$ and $N_{ev} = 35$ respectively for JDQZ case. The change in shape and colour indicates on convergence of eigenvalue pair. What is important to note is that eigenvalue which lies around 7GHz is not calculated.

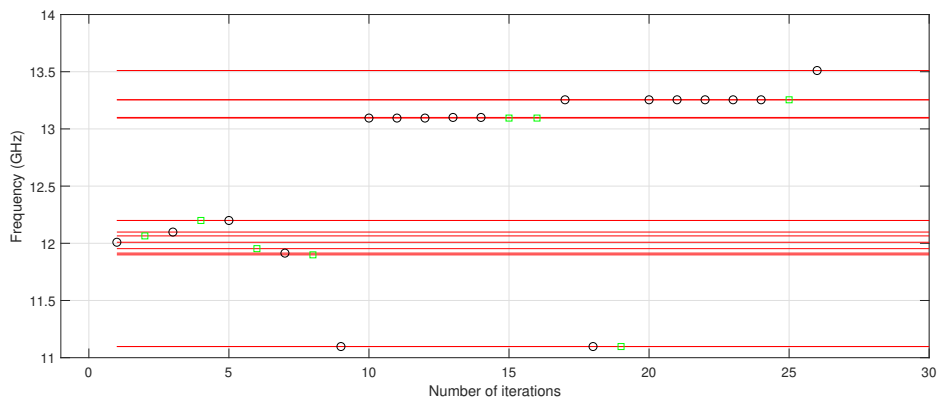


Figure 1: JDQZ case, $N_{ev} = 15$. The change in shape and color of marker indicate on convergence of eigenvalue. Red lines are reference eigenvalues. We can see that 9th eigenvalue needed 5 more iterations to converge (9 to 14 on the x-axis). 10th and 11th eigenvalues required 2 iterations to converge, and 13th eigenvalue required 5 iterations to converge. Rest of eigenvalues converged in 1 iteration.

The Figs.3-4 shows the behaviour of LOBPCG algorithm. What is observed is that many "jumps" occurs in the residuals and eigenvalue errors. In the end of simulation there are still few eigenvalues which error and residuals are at high level.

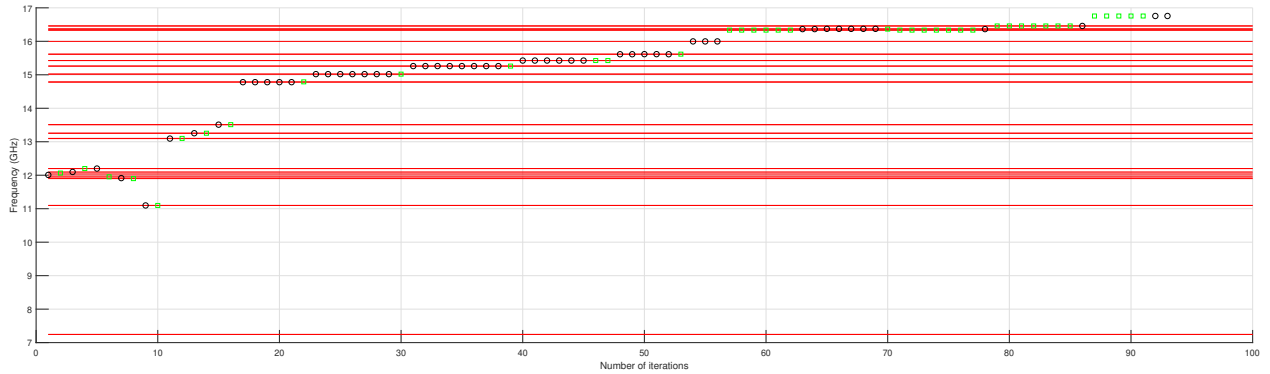


Figure 2: JDQZ case, $N_{ev} = 35$. The change in shape and color of marker indicate on convergence of eigenvalue. Red lines are reference eigenvalues. Two eigenvalues at 11 GHz are very close to each other, however it is not seen on figure.

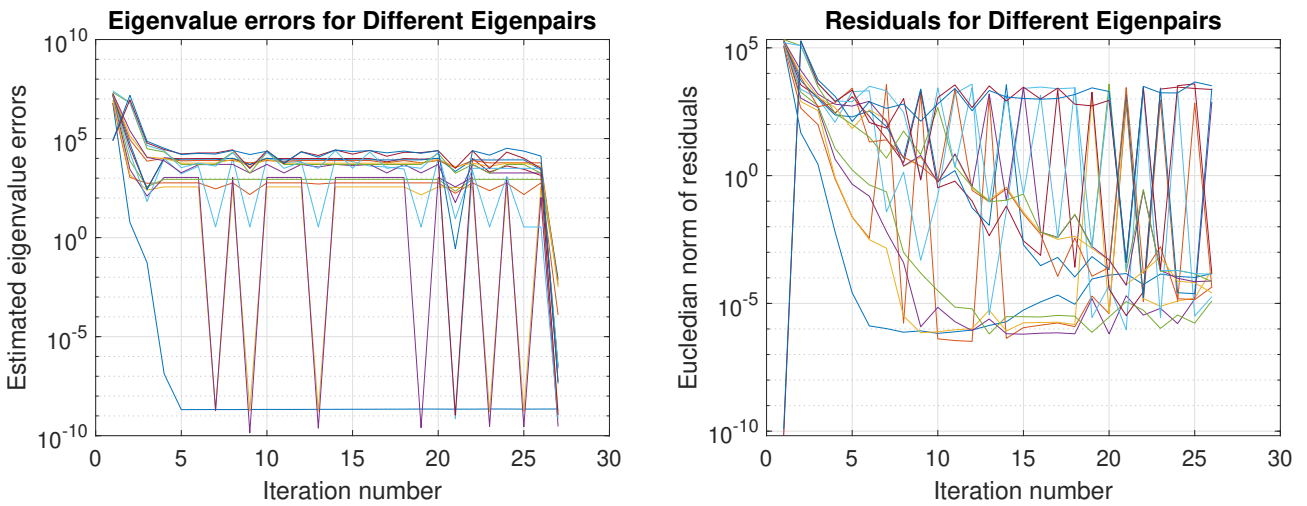


Figure 3: LOBPCG case, $N_{ev} = 15$. Simulation ends after 26 iterations.

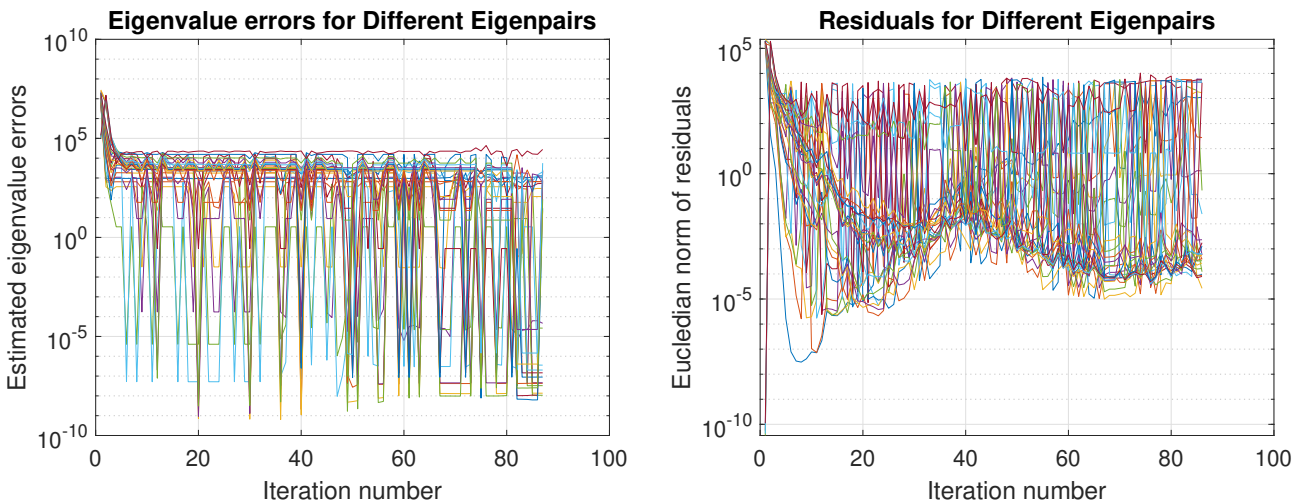


Figure 4: LOBPCG case, $N_{ev} = 35$. Simulation ends after 86 iterations.

2.2. Case 520k variables 32port - PCBLines

In this case eigs seems to be the fastest and the most stable method to compute eigenvalues around central frequency. Resultant times are summarized in Tables.4-8. Structure is shown in Fig.5. Expansion point is placed at 1.45GHz.

LOBPCG algorithm failed in calculation of 15 and 35 eigenvalues (reached 300 iterations and didnt converge). However, while starting space is defined, it starts to converge.

In all cases, number of solves is lowest for eigs and highest for LOBPCG. When number of eigenvalues is high, eigs is the fastest approach.

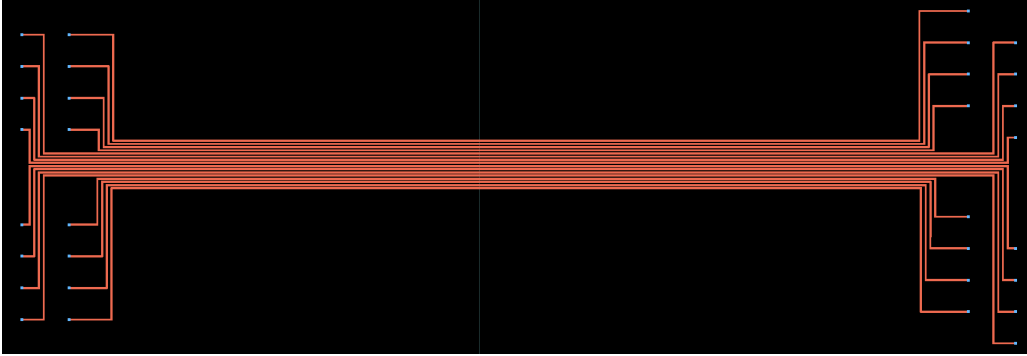


Figure 5: Layout of analyzed case.

| Method | Time | No. Solves |
|--------|------|------------|
| Eigs | 26.6 | 56 |
| LOBPCG | FAIL | FAIL |
| JDQZ | 81.4 | 146 |

Table 4: Time comparison for $N_{ev} = 15$ for PCB Lines. Default settings.

| Method | Time | No. Solves | Tolerance | MaxIterations | Repeatable |
|--------|-------|------------|-----------|---------------|------------|
| Eigs | 43.5 | 89 | 1e-10 | - | Yes |
| LOBPCG | FAIL | FAIL | 1e-4 | 300 | No |
| JDQZ | 193.5 | 227 | 1e-4 | - | Yes |

Table 5: Time comparison for $N_{ev} = 35$ for PCB Lines. Default settings.

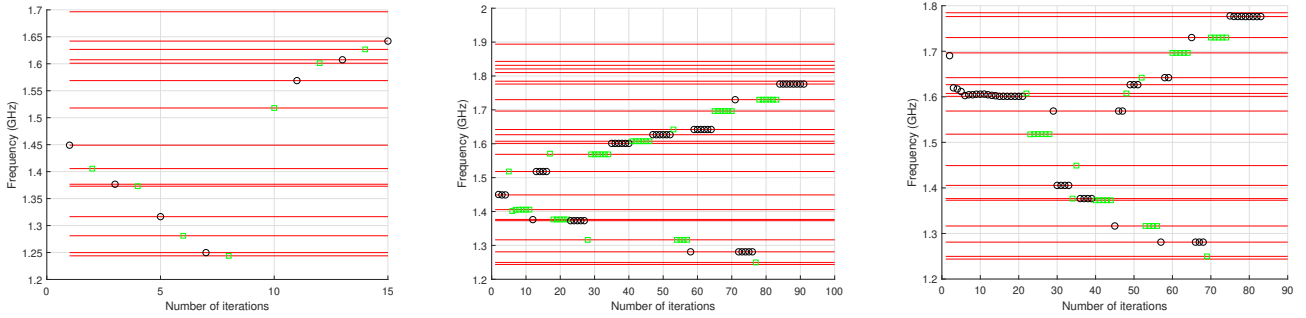


Figure 6: JDQZ convergence plots. On the left starting space is composed of eigenvectors. In the center starting space is composed of field vectors. On the right starting space is composed of eigenvectors obtained by projection onto field solution.

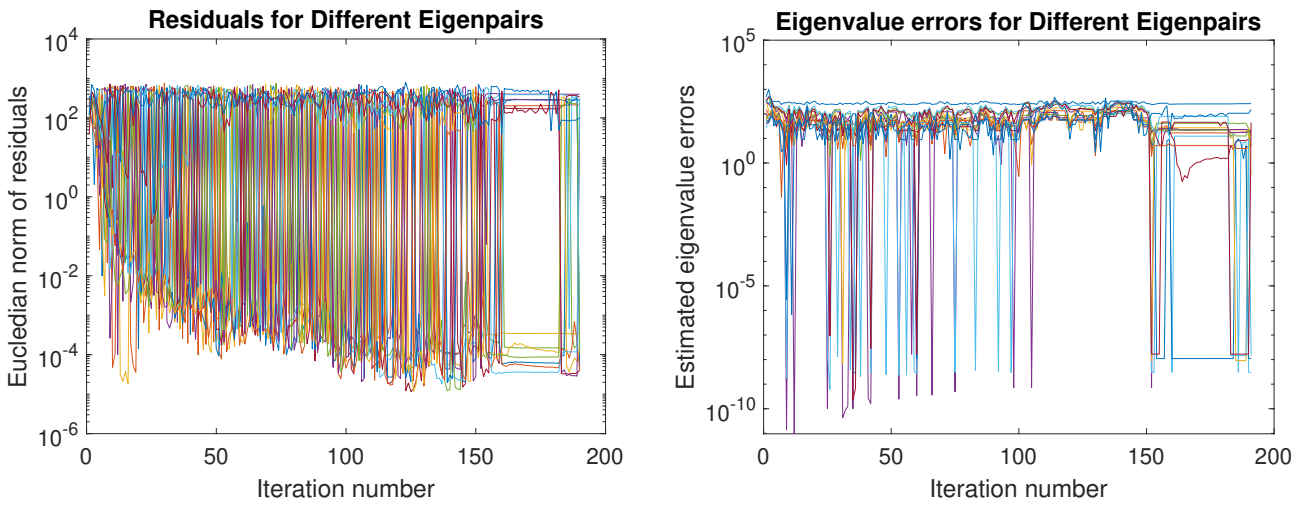


Figure 7: LOBPCG case, starting space is composed of field vectors.

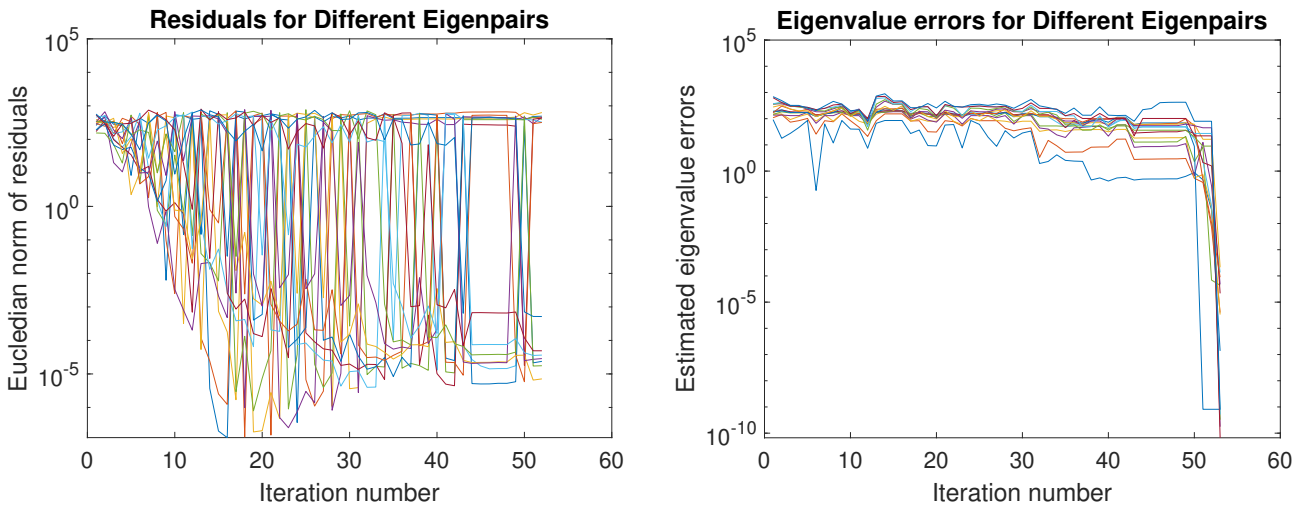


Figure 8: LOBPCG case, starting space is composed of eigenvectors obtained by projection onto field solution.

| Method | Time | No. Solves |
|--------|------|------------|
| LOBPCG | 7.9 | 15 |
| JDQZ | 9.2 | 16 |

Table 6: Time comparison for $N_{ev} = 15$ for PCB Lines. Starting space is composed of eigenvectors.

| Method | Time | No. Solves |
|--------|------|------------|
| LOBPCG | 745 | 3007 |
| JDQZ | 187 | 280 |

Table 7: Time comparison for $N_{ev} = 15$ for PCB Lines. Starting space is composed of field vectors.

| Method | Time | No. Solves |
|--------|------|------------|
| LOBPCG | 250 | 801 |
| JDQZ | 206 | 324 |

Table 8: Time comparison for $N_{ev} = 15$ for PCB Lines. Starting space is composed of eigenvectors obtained by projection onto field solution